

Домашняя работа по алгебре за 9 класс

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СТЕПЕНЬ С РАЦИОНАЛЬНЫМ ПОКАЗАТЕЛЕМ

62.

- 1) $2^3 + (-3)^3 - (-2)^2 + (-1)^5 = 8 + (-27) - (4) + (-1) = -24$;
- 2) $(-7)^2 - (-4)^3 - 3^4 = 49 - (-64) - 81 = 32$;
- 3) $13 \cdot 2^3 - 9 \cdot 2^3 + 2^3 = 2^3 \cdot (13 - 9 + 1) = 8 \cdot 5 = 40$;
- 4) $6 \cdot (-2)^3 - 5 \cdot (-2)^3 - (-2)^3 = -2^3 \cdot (6 - 5 - 1) = 0 \cdot (-2^3) = 0$.

63.

- 1) $\frac{7^2 \cdot 7^{15}}{7^{13}} = \frac{7^{15+2}}{7^{13}} = \frac{7^{17}}{7^{13}} = 7^4$;
- 2) $\frac{5^3 \cdot 5^{10} \cdot 5}{5^4 \cdot 5^{15}} = \frac{5^{10+3+1}}{5^{15+4}} = \frac{5^{14}}{5^{19}} = \frac{1}{5^5} = \left(\frac{1}{5}\right)^5$;
- 3) $\frac{a^2 \cdot a^8 \cdot b^3}{a^9 \cdot b^2} = \frac{a^{2+8} \cdot b^3}{a^9 \cdot b^2} = \frac{a^{10} b^3}{a^9 b^2} = ab$;
- 4) $\frac{c^3 d^5 c^9}{c^{10} d^7} = \frac{c^{12}}{c^{10} d^2} = \frac{c^2}{d^2} = \frac{c^2}{d^2}$.

64.

- 1) $1^{-5} = \frac{1}{1^5} = 1$;
- 2) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$;
- 3) $(-10)^0 = 1$;
- 4) $(-5)^{-2} = \frac{1}{5^2} = \frac{1}{25}$;
- 5) $\left(\frac{1}{2}\right)^4 = \frac{1}{2^4} = \frac{1}{16}$;
- 6) $\left(\frac{3}{7}\right)^{-1} = \frac{7}{3} = 2\frac{1}{3}$.

65.

- 1) $\frac{1}{4^5} = \left(\frac{1}{4}\right)^5 = 4^{-5}$;
- 2) $\frac{1}{21^3} = \left(\frac{1}{21}\right)^3 = 21^{-3}$;
- 3) $\frac{1}{x^7} = \left(\frac{1}{x}\right)^7 = x^{-7}$;
- 4) $\frac{1}{a^9} = \left(\frac{1}{a}\right)^9 = a^{-9}$.

66.

$$1) \left(\frac{10}{3}\right)^{-3} = \frac{3^3}{10^3} = \frac{27}{1000} = 0,027; \quad 2) \left(\frac{-9}{11}\right)^{-2} = \frac{11^2}{9^2} = \frac{121}{81} = 1\frac{40}{80};$$
$$3) (0,2)^{-4} = \left(\frac{1}{5}\right)^{-4} = (5)^4 = 625; \quad 4) (0,5)^{-5} = \left(\frac{1}{2}\right)^{-5} = 2^5 = 32;$$
$$5) -(-17)^{-1} = \frac{1}{17}; \quad 6) -(-13)^{-2} = -\frac{1}{13^2} = -\frac{1}{169}.$$

67.

$$1) 3^{-1} + (-2)^{-2} = \frac{1}{3} + \frac{1}{4} = \frac{3+4}{12} = \frac{7}{12};$$
$$2) \left(\frac{2}{3}\right)^{-3} - 4^{-2} = \frac{3^3}{2^3} - \frac{1}{4^2} = \frac{2 \cdot 27 - 1}{16} = \frac{53}{16} = 3\frac{5}{16};$$
$$3) (0,2)^{-2} + (0,5)^{-5} = 5^2 + 2^5 = 25 + 32 = 57;$$
$$4) (-0,1)^{-3} - (-0,2)^{-3} = -\left(\frac{1}{1000}\right)^{-1} + \left(\frac{1}{125}\right)^{-1} = -1000 + 125 = -875.$$

68.

$$1) 12^{-3} = \frac{1}{12^3} < 1; \quad 2) 21^0 = 1;$$
$$3) (0,6)^{-5} = \left(\frac{5}{3}\right)^5 > 1; \quad 4) \left(\frac{5}{19}\right)^{-4} = \left(\frac{19}{5}\right)^4 > 1.$$

69.

$$1) (x-y)^{-2} = \frac{1}{(x-y)^2}; \quad 2) (x+y)^{-3} = \frac{1}{(x+y)^3};$$
$$3) 3b^{-5}c^8 = \frac{3c^8}{b^5}; \quad 4) 9a^3b^{-4} = \frac{9a^3}{b^4};$$
$$5) a^{-1}b^2c^{-3} = \frac{b^2}{ac^3}; \quad 6) a^2b^{-1}c^{-4} = \frac{a^2}{bc^4}.$$

70.

$$1) \left(\frac{1}{7}\right)^{-3} \cdot \left(\frac{1}{7}\right) = \left(\frac{1}{7}\right)^{-2} = 7^2 = 49;$$
$$2) \left(-\frac{1}{5}\right) \cdot \left(-\frac{1}{5}\right)^{-4} = \left(-\frac{1}{5}\right)^{-3} = (-5)^3 = -125;$$

$$3) 0,3^7 \cdot 0,3^{-10} = 0,3^{-3} = \left(\frac{3}{10}\right)^{-3} = \left(\frac{10}{3}\right)^3 = \frac{1000}{27} = 37\frac{1}{27};$$

$$4) 17^{-5} \cdot 17^3 \cdot 17 = 17^{-1} = \frac{1}{17}.$$

71.

$$1) 9^7 : 9^{10} = 9^{-3} = \frac{1}{9^3} = \frac{1}{729};$$

$$2) (0,2)^2 : (0,2)^{-2} = (0,2)^4 = 0,0016;$$

$$3) \left(\frac{2}{13}\right)^{-12} : \left(\frac{2}{13}\right)^{-10} = \left(\frac{2}{13}\right)^{-2} = \frac{13^2}{2^2} = \frac{169}{4} = 42\frac{1}{4};$$

$$4) \left(\frac{2}{5}\right)^3 : \left(\frac{2}{5}\right)^{-1} = \frac{2^4}{5^4} = \frac{16}{625}.$$

72.

$$1) (a^3)^{-5} = a^{-15};$$

$$2) (b^{-2})^{-4} = b^8;$$

$$3) (a^3)^7 = a^{21};$$

$$4) (b^7)^{-4} = a^{-28}.$$

73.

$$1) (ab^{-2})^3 = a^3 b^{-6} = \frac{a^3}{b^6};$$

$$2) (a^2 b^{-1})^4 = a^8 b^{-4} = \frac{a^8}{b^4};$$

$$3) (2a^2)^{-6} = 2^{-6} a^{-12} = \frac{1}{64a^{12}};$$

$$4) (3a^3)^{-4} = 3^{-4} a^{-12} = \frac{1}{81a^{12}}.$$

74.

$$1) \left(\frac{a^8}{b^7}\right)^{-2} = \frac{a^{-16}}{b^{-14}} = \frac{b^{14}}{a^{16}};$$

$$2) \left(\frac{m^{-4}}{n^{-5}}\right)^{-3} = \frac{m^{12}}{n^{15}};$$

$$3) \left(\frac{2x^6}{3y^{-4}}\right)^2 = \frac{2^2 x^{12} y^8}{3^2} = \frac{4x^{12} y^8}{9};$$

$$4) \left(\frac{-4yx^{-5}}{z^3}\right)^3 = \frac{-64y^3 x^{-15}}{z^9} = -\frac{64y^3}{z^9 x^{15}};$$

75.

$$1) (x^2 y^{-2} - 4y^{-2}) \cdot \left(\frac{1}{y}\right)^{-2} = (x^2 - 4) \cdot y^{-2} \cdot y^2 = x^2 - 4,$$

если $x = 5$, то $x^2 = 25$ и $25 - 4 = 21$;

$$2) \left((a^2 b^{-1})^4 - a^0 b^4 \right) : \frac{a^4 - b^4}{b^2} = \left(\frac{a^8}{b^4} - b^4 \right) \cdot \frac{b^2}{a^4 - b^4} = \\ = \frac{(a^8 - b^8)}{b^4} \cdot \frac{b^2}{(a^4 - b^4)} = \frac{(a^4 - b^4)(a^4 + b^4)}{b^2 \cdot (a^4 - b^4)} = \frac{a^4 + b^4}{b^2};$$

если $a = 2$, $b = -3$, то $a^4 = 16$, $b^4 = 81$, $b^2 = 9$ и $\frac{16 + 81}{9} = \frac{97}{9} = 10\frac{7}{9}$.

76.

$$1) 200000^4 = (2 \cdot 10^5)^4 = 2^4 \cdot 10^{20} = 16 \cdot 10^{20} = 1,6 \cdot 10^{21};$$

$$2) 0,0003^3 = (3 \cdot 10^{-4})^3 = 3^3 \cdot 10^{-12} = 27 \cdot 10^{-12} = 2,7 \cdot 10^{-11};$$

$$3) 4000^{-2} = (4 \cdot 10^3)^{-2} = 0,0625 \cdot 10^{-6} = 6,25 \cdot 10^{-8};$$

$$4) 0,002^{-3} = (2 \cdot 10^{-3})^{-3} = 2^{-3} \cdot 10^9 = 0,125 \cdot 10^9 = 1,25 \cdot 10^8.$$

77.

$$1) 0,0000087 = 8,7 \cdot 10^{-6};$$

$$2) 0,00000005086 = 5,086 \cdot 10^{-8};$$

$$3) \frac{1}{125} = 0,008 = 8 \cdot 10^{-3};$$

$$4) \frac{1}{625} = 0,0016 = 1,6 \cdot 10^{-3}.$$

78, 79, 80.

$$3 \cdot 10^{-3} \text{мм} = \frac{3}{1000} \text{мм} = 0,003 \text{мм}; 0,00000000001 \text{с} = 10^{-11} \text{с};$$

$$10^{-4} \text{мм} = 0,0001 \text{мм}.$$

81.

$$1) \frac{a^8 a^{-7}}{a^{-2}} = a^{8-7+2} = a^3,$$

если $a = 0,8$, то $a^3 = 0,512$;

$$2) \frac{a^{15} a^3}{a^{13}} = a^{15+3-13} = a^5,$$

если $a = \frac{1}{2}$, то $a^5 = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$.

82.

$$1) ((-20)^7)^{-7} : ((-20)^{-6})^8 + 2^{-2} = ((-20)^{-49} : (-20)^{-48}) + \frac{1}{4} = \\ = -\frac{1}{20} + \frac{1}{4} = \frac{-1+5}{20} = \frac{1}{5};$$

$$2) ((-17)^{-4})^{-6} : ((-17)^{-13})^{-2} - \left(\frac{1}{17}\right)^{-2} = (-17)^{24} : (-17)^{26} - \\ - \left(\frac{1}{17}\right)^2 = \left(-\frac{1}{17}\right)^2 - \left(\frac{1}{17}\right)^2 = \frac{1}{17^2} - \frac{1}{17^2} = 0.$$

83.

$$1) (1,3)^{-118} \cdot (1,3)^{127} = (1,3)^9 \approx 10,6;$$

$$2) (0,87)^{-74} : (0,87)^{-57} = (0,87)^{-74+57} = (0,87)^{-17} \approx 10,67;$$

$$3) \left(\frac{17}{19}\right)^{-47} : \left(\frac{17}{19}\right)^{-26} = \left(\frac{17}{19}\right)^{-21} = \left(\frac{19}{17}\right)^{21} \approx 10,34;$$

$$4) \left(\frac{23}{21}\right)^{56} \cdot \left(\frac{23}{21}\right)^{-25} = \left(\frac{23}{21}\right)^{31} \approx 16,78.$$

84.

$$1) (786^{-7})^4 = 786^{-28} = 5,8 \cdot 10^{-62};$$

$$2) (923^3)^{-6} = 923^{-18} = 4,23 \cdot 10^{-54};$$

$$3) (1,76)^{-8} \cdot (35,4)^{-8} = (62,3)^{-8} = 2,07 \cdot 10^{-14};$$

$$4) (0,47)^{-5} : (7,81)^{-5} = (0,47 : 7,81)^{-5} = 1,27 \cdot 10^6.$$

85.

$$1) V = (1,54 \cdot 10^{-4})^3 = 3,65 \cdot 10^{-12} \text{ мм}^3;$$

$$2) V = (3,18 \cdot 10^5)^3 = 3,21 \cdot 10^{15} \text{ км}^3.$$

86.

$$1) (a^{-3} + b^{-3}) \cdot (a^{-2} - b^{-2})^{-1} \cdot (a^{-2} - a^{-1}b^{-1} + b^{-2})^{-1} = \left(\frac{1}{a^3} + \frac{1}{b^3}\right) \times \\ \times \left(\frac{1}{a^2} - \frac{1}{b^2}\right)^{-1} \cdot \left(\frac{1}{a^2} - \frac{1}{ab} + \frac{1}{b^2}\right)^{-1} = \frac{b^3 + a^3}{a^3 b^3} \cdot \frac{a^2 b^2}{b^2 - a^2} \cdot \frac{a^2 b^2}{b^2 - ab + a^2} = \\ = \frac{(b^3 + a^3) \cdot a^4 b^4}{a^3 b^3 \cdot (b-a)(b+a)(b^2 - ab + a^2)} = \frac{ab(b^3 + a^3)}{(b-a)(a^3 + b^3)} = \frac{ab}{b-a};$$

$$\begin{aligned}
 & 2) (a^{-2}b - ab^{-2}) \cdot (a^{-2} + a^{-1}b^{-1} + b^{-2})^{-1} = \\
 & = \left(\frac{b}{a^2} - \frac{a}{b^2} \right) \cdot \left(\frac{1}{a^2} + \frac{1}{ab} + \frac{1}{b^2} \right)^{-1} = \\
 & = \frac{b^3 - a^3}{a^2b^2} \cdot \frac{a^2b^2}{b^2 + ab + a^2} = \frac{(b-a)(b^2 + ab + a^2)}{b^2 + ab + a^2} = b - a.
 \end{aligned}$$

87.

$$1) \sqrt[3]{1} = 1; \quad \sqrt[3]{0} = 0; \quad \sqrt[3]{16} = \sqrt[3]{4^2} = 4; \quad \sqrt[3]{169} = \sqrt[3]{13^2} = 13;$$

$$\sqrt{\frac{1}{289}} = \sqrt{\left(\frac{1}{17}\right)^2} = \frac{1}{17};$$

$$2) \sqrt[3]{1} = 1; \quad \sqrt[3]{0} = 0; \quad \sqrt[3]{125} = \sqrt[3]{5^3} = 5; \quad \sqrt[3]{\frac{1}{27}} = \sqrt[3]{\frac{1}{3^3}} = \frac{1}{3};$$

$$\sqrt[3]{0,027} = \sqrt[3]{(0,3)^3} = 0,3; \quad \sqrt[3]{0,064} = \sqrt[3]{(0,4)^3} = 0,4$$

$$3) \sqrt[4]{0} = 0; \quad \sqrt[4]{1} = 1; \quad \sqrt[4]{16} = \sqrt[4]{2^4} = 2; \quad \sqrt[4]{\frac{16}{81}} = \sqrt[4]{\left(\frac{2}{3}\right)^4} = \frac{2}{3};$$

$$\sqrt[4]{\frac{256}{625}} = \sqrt[4]{\left(\frac{4}{5}\right)^4} = \frac{4}{5}; \quad \sqrt[4]{0,0016} = \sqrt[4]{(0,2)^4} = 0,2.$$

88.

$$1) \sqrt[6]{36^3} = \sqrt[6]{(6^2)^3} = \sqrt[6]{6^6} = 6; \quad 2) \sqrt[12]{64^2} = \sqrt[12]{(2^6)^2} = \sqrt[12]{2^{12}} = 2;$$

$$3) \sqrt[4]{\left(\frac{1}{25}\right)^2} = \sqrt[4]{\left(\frac{1}{5}\right)^4} = \frac{1}{5}; \quad 4) \sqrt[8]{225^4} = \sqrt[8]{(15^2)^4} = \sqrt[8]{15^8} = 15.$$

89.

$$1) \sqrt[3]{10^6} = 10^2 = 100; \quad 2) \sqrt[3]{3^{12}} = 3^4 = 81;$$

$$3) \sqrt[4]{\left(\frac{1}{2}\right)^{12}} = \left(\frac{1}{2}\right)^3 = \frac{1}{2^3} = \frac{1}{8};$$

$$4) \sqrt[4]{\left(\frac{1}{3}\right)^{16}} = \left(\frac{1}{3}\right)^4 = \frac{1}{3^4} = \frac{1}{81}.$$

90.

1) $\sqrt[3]{-8} = -2$;

2) $\sqrt[15]{-1} = -1$;

3) $\sqrt[3]{-\frac{1}{27}} = -\frac{1}{\sqrt[3]{27}} = -\frac{1}{3}$;

4) $\sqrt[5]{-1024} = -\sqrt[5]{4^5} = -4$;

5) $\sqrt[3]{-34^3} = -34$;

6) $\sqrt[7]{-8^7} = -8$.

91.

1) $x^4 = 81$; $x = \pm \sqrt[4]{81} = \pm 3$; $x_1 = 3$; $x_2 = -3$;

2) $x^5 = -\frac{1}{32}$; $x = \sqrt[5]{-\frac{1}{32}} = \sqrt[5]{\left(-\frac{1}{2}\right)^5} = -\frac{1}{2}$;

3) $5x^5 = -160$; $x^5 = -32$; $x = \sqrt[5]{-32} = -2$.

4) $2x^6 = 128$; $x^6 = 64$; $x = \pm \sqrt[6]{64} = \pm 2$; $x_1 = 2$, $x_2 = -2$.

92.

1) $\sqrt[6]{2x-3}$ — имеет смысл, если

$2x - 3 \geq 0$, тогда $2x \geq 3$, $x \geq \frac{3}{2}$,

$x \geq 1,5$.

Ответ: $x \in [1,5; +\infty)$.

2) $\sqrt[3]{x+3}$ — имеет смысл для любого x .

3) $\sqrt[3]{2x^2-x-1}$ — имеет смысл для любого x .

4) $\sqrt[4]{\frac{2-3x}{2x-4}}$ — имеет смысл, если: $\frac{2-3x}{2x-4} \geq 0$, т.е. $\begin{cases} 2-3x \geq 0 \\ 2x-4 > 0 \end{cases}$

или $\begin{cases} 2-3x \leq 0 \\ 2x-4 < 0 \end{cases}$; $\begin{cases} x \leq \frac{2}{3} \\ x > 2 \end{cases}$ или $\begin{cases} x \geq \frac{2}{3} \\ x < 2 \end{cases}$, поэтому $\begin{cases} x \geq \frac{2}{3} \\ x < 2 \end{cases}$

Ответ: $x \in [\frac{2}{3}; 2)$.

93.

1) $\sqrt[3]{-125} + \frac{1}{8}\sqrt[6]{64} = \sqrt[3]{(-5)^3} + \frac{1}{8}\sqrt[6]{2^6} = -5 + \frac{1}{8} \cdot 2 = -5 + \frac{1}{4} = -4\frac{3}{4}$;

2) $\sqrt[5]{32} - 0,5 \cdot \sqrt[3]{-216} = \sqrt[5]{2^5} - \frac{1}{2}\sqrt[3]{(-6)^3} = 2 + \frac{6}{2} = 5$;

$$3) -\frac{1}{3}\sqrt[4]{81} + \sqrt[4]{625} = -\frac{1}{3}\sqrt[4]{3^4} + \sqrt[4]{5^4} = -\frac{1}{3} \cdot 3 + 5 = -1 + 5 = 4;$$

$$4) \sqrt[3]{-1000} - \frac{1}{4}\sqrt[4]{256} = \sqrt[3]{(-10)^3} - \frac{1}{4}\sqrt[4]{4^4} = -10 - 1 = -11;$$

$$5) \sqrt[4]{0,0001} - 2 \cdot \sqrt{0,25} + \sqrt[5]{-\frac{1}{32}} = \sqrt[4]{(0,1)^4} - 2\sqrt{0,5^2} + \sqrt[5]{\left(-\frac{1}{2}\right)^5} =$$

$$= 0,1 - 1 - \frac{1}{2} = -1,4;$$

$$6) \sqrt[5]{\frac{1}{243}} + \sqrt[3]{-0,001} - \sqrt[4]{0,0016} = \frac{1}{3} - 0,1 - 0,2 = \frac{1}{3} - 0,3 = \frac{1}{3} - \frac{3}{10} = \frac{10-9}{30} = \frac{1}{30}.$$

94.

$$1) \sqrt{9+\sqrt{17}} \cdot \sqrt{9-\sqrt{17}} = \sqrt{81-17} = \sqrt{64} = 8;$$

$$2) \left(\sqrt{3+\sqrt{5}} - \sqrt{3-\sqrt{5}} \right)^2 = 3 + \sqrt{5} - 2\sqrt{9-5} + 3 - \sqrt{5} = 6 - 4 = 2;$$

$$3) \left(\sqrt{5+\sqrt{21}} + \sqrt{5-\sqrt{21}} \right)^2 = 5 + \sqrt{21} + 2\sqrt{25-21} + 5 - \sqrt{21} =$$

$$= 10 + 4 = 14;$$

$$4) \frac{\sqrt{3+\sqrt{2}}}{\sqrt{3-\sqrt{2}}} - \frac{\sqrt{3-\sqrt{2}}}{\sqrt{3+\sqrt{2}}} = \frac{(\sqrt{3+\sqrt{2}})^2 - (\sqrt{3-\sqrt{2}})^2}{3-2} =$$

$$= \frac{3+2\sqrt{6}+2-3+2\sqrt{6}-2}{3-2} = \frac{2\sqrt{6}+2\sqrt{6}}{1} = 4\sqrt{6}.$$

95.

$$1) \sqrt[3]{(x-2)^3} = x-2 \text{ — для любого } x.$$

$$2) \text{ т.к. } \sqrt{(3-x)^6} \geq 0, \text{ то при } x < 3 \sqrt{(3-x)^6} = (3-x)^3$$

$$\text{и при } x \geq 3 \sqrt{(3-x)^6} = -(3-x)^3 = (x-3)^3.$$

96.

$$1987 < \sqrt{n} < 1988; 1987^2 < n < 1988^2, \text{ отсюда}$$

$$3948169 < n < 3952144.$$

Найдем, сколько натуральных чисел между ними
 $3952144 - 3948169 = 3975$, а т.к. $n < 3952144$, то таких чисел 3974.

Ответ: 3974 числа.

97.

- 1) $\sqrt[3]{343 \cdot 0,125} = \sqrt[3]{7^3 \cdot (0,5)^3} = \sqrt[3]{(7 \cdot 0,5)^3} = \sqrt[3]{(3,5)^3} = 3,5;$
- 2) $\sqrt[3]{864 \cdot 216} = \sqrt[3]{3^3 \cdot 2^5 \cdot 2^3 \cdot 3^3} = 3^2 \cdot 2^2 \cdot \sqrt[3]{2^2} = 9 \cdot 4\sqrt[3]{4} = 36 \cdot \sqrt[3]{4};$
- 3) $\sqrt[4]{256 \cdot 0,0081} = \sqrt[4]{2^8 \cdot (0,3)^4} = 2^2 \cdot 0,3 = 4 \cdot 0,3 = 1,2;$
- 4) $\sqrt[5]{32 \cdot 100000} = \sqrt[5]{2^5 \cdot 10^5} = 2 \cdot 10 = 20.$

98.

- 1) $\sqrt[3]{5^3 \cdot 7^3} = \sqrt[3]{(5 \cdot 7)^3} = \sqrt[3]{35^3} = 35;$
- 2) $\sqrt[4]{11^4 \cdot 3^4} = \sqrt[4]{(11 \cdot 3)^4} = \sqrt[4]{33^4} = 33;$
- 3) $\sqrt[5]{(0,2)^5 \cdot 8^5} = \sqrt[5]{(0,2 \cdot 8)^5} = \sqrt[5]{1,6^5} = 1,6;$
- 4) $\sqrt[7]{\left(\frac{1}{3}\right)^7 \cdot 21^7} = \sqrt[7]{\left(\frac{1}{3} \cdot 21\right)^7} = \sqrt[7]{7^7} = 7.$

99.

- 1) $\sqrt[3]{2} \cdot \sqrt[3]{500} = \sqrt[3]{1000} = \sqrt[3]{10^3} = 10;$
- 2) $\sqrt[3]{0,2} \cdot \sqrt[3]{0,04} = \sqrt[3]{0,008} = \sqrt[3]{0,2^3} = 0,2;$
- 3) $\sqrt[4]{324} \cdot \sqrt[4]{4} = \sqrt[4]{81 \cdot 16} = \sqrt[4]{3^4 \cdot 2^4} = \sqrt[4]{6^4} = 6;$
- 4) $\sqrt[3]{2} \cdot \sqrt[5]{16} = \sqrt[5]{32} = \sqrt[5]{2^5} = 2.$

100.

- 1) $\sqrt[5]{3^{10} \cdot 2^{15}} = 3^2 \cdot 2^3 = 9 \cdot 8 = 72;$
- 2) $\sqrt[3]{2^3 \cdot 5^6} = 2 \cdot 5^2 = 2 \cdot 25 = 50;$
- 3) $\sqrt[4]{3^{12} \cdot \left(\frac{1}{3}\right)^8} = 3^3 \cdot \left(\frac{1}{3}\right)^2 = \frac{27}{9} = 3;$
- 4) $\sqrt[10]{4^{30} \cdot \left(\frac{1}{2}\right)^{20}} = 4^3 \cdot \left(\frac{1}{2}\right)^2 = \frac{64}{4} = 16.$

(101 – 102)

- 1) $\sqrt[3]{64 \cdot x^3 \cdot z^6} = 4xz^2;$
- 2) $\sqrt[4]{a^8 \cdot b^{12}} = a^2b^3;$
- 3) $\sqrt[5]{32 \cdot x^{10} \cdot y^{20}} = 2x^2y^4;$
- 4) $\sqrt[6]{a^{12}b^{18}} = a^2b^3.$

102.

- 1) $\sqrt[3]{2ab^2} \cdot \sqrt[3]{4a^2b} = \sqrt[3]{2^3 a^3 b^3} = 2ab$; 2) $\sqrt[4]{3a^2b^3} \cdot \sqrt[4]{27a^2b} = \sqrt[4]{3^4 a^4 b^4} = 3ab$;
 3) $\sqrt[4]{\frac{ab}{c}} \cdot \sqrt[4]{\frac{a^3c}{b}} = \sqrt[4]{\frac{a^4bc}{bc}} = a$; 4) $\sqrt[3]{\frac{16a}{b^2}} \cdot \sqrt[3]{\frac{1}{2ba}} = \sqrt[3]{\frac{16a}{2ab^3}} = \frac{2}{b}$.

103.

- 1) $\sqrt[3]{\frac{64}{125}} = \sqrt[3]{\frac{4^3}{5^3}} = \sqrt[3]{\left(\frac{4}{5}\right)^3} = \frac{4}{5}$; 2) $\sqrt[4]{\frac{16}{81}} = \sqrt[4]{\left(\frac{2}{3}\right)^4} = \frac{2}{3}$;
 3) $\sqrt[3]{3\frac{3}{8}} = \sqrt[3]{\frac{27}{8}} = \sqrt[3]{\left(\frac{3}{2}\right)^3} = \frac{3}{2}$; 4) $\sqrt[5]{7\frac{19}{32}} = \sqrt[5]{\frac{243}{32}} = \sqrt[5]{\left(\frac{3}{2}\right)^5} = \frac{3}{2}$.

104.

- 1) $\sqrt[4]{324} : \sqrt[4]{4} = \sqrt[4]{\frac{324}{4}} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$;
 2) $\sqrt[3]{128} : \sqrt[3]{2000} = \sqrt[3]{\frac{128}{2 \cdot 10^3}} = \sqrt[3]{\frac{64}{1000}} = \sqrt[3]{\left(\frac{4}{10}\right)^3} = \frac{4}{10} = \frac{2}{5}$;
 3) $\frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \sqrt[3]{\frac{16}{2}} = \sqrt[3]{8} = \sqrt[3]{2^3} = 2$; 4) $\frac{\sqrt[5]{256}}{\sqrt[5]{8}} = \sqrt[5]{\frac{256}{8}} = \sqrt[5]{32} = \sqrt[5]{2^5} = 2$;
 5) $(\sqrt{20} - \sqrt{45}) : \sqrt{5} = \sqrt{\frac{20}{5}} - \sqrt{\frac{45}{5}} = \sqrt{4} - \sqrt{9} = 2 - 3 = -1$;
 6) $(\sqrt[3]{625} - \sqrt[3]{5}) : \sqrt[3]{5} = \sqrt[3]{\frac{625}{5}} - \sqrt[3]{\frac{5}{5}} = \sqrt[3]{125} - 1 = \sqrt[3]{5^3} - 1 = 5 - 1 = 4$.

105.

- 1) $\sqrt[5]{a^6b^7} : \sqrt[5]{ab^2} = \sqrt[5]{\frac{a^6b^7}{ab^2}} = \sqrt[5]{a^5b^5} = ab$;
 2) $\sqrt[3]{81x^4y} : \sqrt[3]{3xy} = \sqrt[3]{\frac{81x^4y}{3xy}} = \sqrt[3]{27x^3} = \sqrt[3]{3^3x^3} = 3x$;
 3) $\sqrt[3]{\frac{3x}{y^2}} : \sqrt[3]{\frac{y}{9x^2}} = \sqrt[3]{\frac{27x^3}{y^3}} = \frac{3x}{y}$;
 4) $\sqrt[4]{\frac{2b}{a^3}} : \sqrt[4]{\frac{a}{8b^3}} = \sqrt[4]{\frac{16b^4}{a^4}} = \frac{2b}{a}$.

106.

$$1) \left(\sqrt[6]{7^3}\right)^2 = \sqrt[6]{7^6} = 7; \quad 2) \left(\sqrt[6]{9}\right)^{-3} = 9^{-\frac{3}{6}} = 9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3};$$

$$3) \left(\sqrt[10]{32}\right)^2 = 32^{\frac{2}{10}} = 32^{\frac{1}{5}} = \sqrt[5]{32} = \sqrt[5]{2^5} = 2;$$

$$4) \left(\sqrt[8]{16}\right)^4 = 16^{\frac{4}{8}} = 16^{\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}} = \frac{1}{4}.$$

107.

$$1) \sqrt[3]{\sqrt[3]{729}} = \sqrt[6]{3^6} = 3; \quad 2) \sqrt{\sqrt[4]{1024}} = \sqrt[8]{2^{10}} = 2^{\frac{5}{4}} = 4\sqrt{2};$$

$$3) \sqrt[3]{\sqrt[3]{9}} \cdot \sqrt[9]{3^7} = \sqrt[3]{3^2 \cdot 3^7} = \sqrt[3]{3^9} = 3;$$

$$4) \sqrt[4]{\sqrt[3]{25}} \cdot \sqrt[6]{5^5} = \sqrt[12]{25} \cdot \sqrt[12]{5^{10}} = \sqrt[12]{5^2 \cdot 5^{10}} = \sqrt[12]{5^{12}} = 5.$$

108.

$$1) \left(\sqrt[3]{x}\right)^6 = x^{\frac{6}{3}} = x^2; \quad 2) \left(\sqrt[3]{y^2}\right)^3 = \sqrt[3]{y^6} = y^{\frac{6}{3}} = y^2;$$

$$3) \left(\sqrt{a} \cdot \sqrt[3]{b}\right)^6 = a^{\frac{6}{2}} \cdot b^{\frac{6}{3}} = a^3 b^2;$$

$$4) \left(\sqrt[3]{a^2} \cdot \sqrt[4]{b^3}\right)^{12} = a^{\frac{24}{3}} \cdot b^{\frac{36}{4}} = a^8 b^9;$$

$$5) \left(\sqrt[3]{\sqrt{a^2 b}}\right)^6 = \left(a^{\frac{2}{6}} \cdot b^{\frac{1}{6}}\right)^6 = a^2 b;$$

$$6) \left(\sqrt[3]{\sqrt[4]{27a^3}}\right)^4 = \left(27^{\frac{1}{12}} \cdot a^{\frac{3}{12}}\right)^4 = \sqrt[3]{27a^3} = \sqrt[3]{(3a)^3} = 3a.$$

109.

$$1) \sqrt[3]{\frac{3}{2}} \cdot \sqrt[3]{2\frac{1}{4}} = \sqrt[3]{\frac{3}{2} \cdot \frac{9}{4}} = \sqrt[3]{\left(\frac{3}{2}\right)^3} = \frac{3}{2};$$

$$2) \sqrt[4]{\frac{3}{4}} \cdot \sqrt[4]{6\frac{3}{4}} = \sqrt[4]{\frac{3}{4} \cdot \frac{27}{4}} = \sqrt[4]{\left(\frac{3}{2}\right)^4} = \frac{3}{2};$$

$$3) \sqrt[4]{15 \frac{5}{8}} : \sqrt[4]{\frac{2}{5}} = \sqrt[4]{\frac{125}{8} \cdot \frac{5}{2}} = \sqrt[4]{\left(\frac{5}{2}\right)^4} = \frac{5}{2};$$

$$4) \sqrt[3]{11 \frac{1}{4}} : \sqrt[3]{3 \frac{1}{3}} = \sqrt[3]{\frac{45}{4} \cdot \frac{3}{10}} = \sqrt[3]{\frac{27}{8}} = \sqrt[3]{\left(\frac{3}{2}\right)^3} = \frac{3}{2};$$

$$5) \left(\sqrt[3]{\sqrt{27}}\right)^2 = \sqrt[6]{3^6} = 3; 6) \left(\sqrt{\sqrt[3]{16}}\right)^3 = \sqrt[6]{2^{12}} = 2^2 = 4.$$

110.

$$1) \sqrt[3]{\frac{ab^2}{c}} \cdot \sqrt[3]{\frac{a^5b}{c^2}} = \sqrt[3]{\frac{a^6b^3}{c^3}} = \frac{a^2b}{c};$$

$$2) \sqrt[5]{\frac{8a^3}{b^2}} \cdot \sqrt[5]{\frac{4a^7}{b^3}} = \sqrt[5]{\frac{2^5 a^{10}}{b^5}} = \frac{2a^2}{b};$$

$$3) \frac{\sqrt[4]{a^2b^2c} \cdot \sqrt[4]{a^3b^3c^2}}{\sqrt[4]{abc^3}} = \sqrt[4]{\frac{a^2b^2c \cdot a^3b^3c^2}{abc^3}} = \sqrt[4]{a^4b^4} = ab;$$

$$4) \frac{\sqrt[3]{2a^4b} \cdot \sqrt[3]{4ab}}{2b\sqrt[3]{a^2b^2}} = \frac{1}{2b} \sqrt[3]{\frac{2^3 a^5b^2}{a^2b^2}} = \frac{\sqrt[3]{8a^3}}{2b} = \frac{2a}{2b} = \frac{a}{b};$$

$$5) \left(\sqrt[5]{a^3}\right)^5 \cdot \left(\sqrt[3]{b^2}\right)^3 = a^3b^2;$$

$$6) \left(\sqrt[4]{a^3b^3}\right)^4 : \left(\sqrt[3]{ab^2}\right)^3 = \frac{a^3b^3}{ab^2} = a^2b.$$

111.

$$1) \frac{\sqrt[3]{49} \cdot \sqrt[3]{112}}{\sqrt[3]{250}} = \sqrt[3]{\frac{49 \cdot 56}{125}} = \sqrt[3]{\frac{7^2 \cdot 7 \cdot 8}{5^3}} =$$

$$= \sqrt[3]{\frac{7^3 \cdot 2^3}{5^3}} = \sqrt[3]{\left(\frac{14}{5}\right)^3} = 2 \frac{4}{5};$$

$$2) \frac{\sqrt[4]{54} \cdot \sqrt[4]{120}}{\sqrt[4]{5}} = \sqrt[4]{54 \cdot 24} = \sqrt[4]{27 \cdot 2 \cdot 8 \cdot 3} = \sqrt[4]{2^4 \cdot 3^4} = 2 \cdot 3 = 6;$$

$$3) \frac{\sqrt[4]{32}}{\sqrt[4]{2}} + \sqrt[6]{27^2} - \sqrt{\sqrt[3]{64}} = \sqrt[4]{\frac{32}{2}} + 3 - \sqrt[6]{2^6} = \sqrt[4]{16} + 3 - 2 = 2 + 1 = 3;$$

$$4) \sqrt[3]{3\frac{3}{8}} + \sqrt[4]{18} \cdot \sqrt[4]{4\frac{1}{2}} - \sqrt{\sqrt{256}} = \sqrt[3]{\frac{27}{8}} + \sqrt[4]{\frac{2 \cdot 3^2 \cdot 3^2}{2}} - \sqrt[4]{4^4} =$$

$$= \frac{3}{2} + 3 - 4 = \frac{1}{2};$$

$$5) \sqrt[3]{11 - \sqrt{57}} \cdot \sqrt[3]{11 + \sqrt{57}} = \sqrt[3]{11^2 - 57} = \sqrt[3]{121 - 57} = \sqrt[3]{64} = 4;$$

$$6) \sqrt[4]{17 - \sqrt{33}} \cdot \sqrt[4]{17 + \sqrt{33}} = \sqrt[4]{17^2 - 33} = \sqrt[4]{256} = \sqrt[4]{4^4} = 4.$$

112.

$$1) \sqrt[3]{2ab} \cdot \sqrt[3]{4a^2b} \cdot \sqrt[3]{27b} = \sqrt[3]{2^3 \cdot a^3 b^3 \cdot 3^3} = 2 \cdot 3 \cdot ba = 6ab;$$

$$2) \sqrt[4]{abc} \cdot \sqrt[4]{a^3b^2c} \cdot \sqrt[4]{b^5c^2} = \sqrt[4]{a^4b^8c^4} = ab^2c;$$

$$3) \frac{\sqrt[5]{a^3b^2} \cdot \sqrt[5]{3a^2b^3}}{\sqrt[5]{3ab}} = \frac{\sqrt[5]{a^5b^5 \cdot 3}}{\sqrt[5]{3ab}} = \sqrt[5]{a^4b^4};$$

$$4) \frac{\sqrt[4]{8x^2y^5} \cdot \sqrt[4]{4x^3y}}{\sqrt[4]{2xy^2}} = \sqrt[4]{\frac{16 \cdot x^5y^6}{xy^2}} = \sqrt[4]{16x^4y^4} = 2xy.$$

113.

$$1) \sqrt[3]{\sqrt[3]{a^{18}}} + \left(\sqrt[3]{\sqrt[3]{a^4}} \right)^3 = a^{\frac{18}{9}} + a^{\frac{12}{6}} = a^2 + a^2 = 2a^2;$$

$$2) \left(\sqrt[3]{\sqrt[3]{x^2}} \right)^3 + 2 \left(\sqrt[4]{\sqrt{x}} \right)^8 = x^{\frac{6}{6}} + 2x^{\frac{8}{8}} = x + 2x = 3x;$$

$$3) 2\sqrt{\sqrt{a^4b^8}} - \left(\sqrt[3]{\sqrt{a^3b^6}} \right)^2 = 2a^{\frac{4}{4}}b^{\frac{8}{4}} - a^{\frac{6}{6}}b^{\frac{12}{6}} = 2ab^2 - ab^2 = ab^2;$$

$$4) \sqrt[3]{\sqrt{x^6y^{12}}} - \left(\sqrt[5]{xy^2} \right)^5 = \sqrt[6]{x^6y^{12}} - xy^2 = xy^2 - xy^2 = 0;$$

$$5) \left(\sqrt[4]{\sqrt{x^8y^2}} \right)^4 - \left(\sqrt[4]{x^2y^8} \right)^2 = \sqrt[8]{x^{32}y^8} - \sqrt[4]{x^4y^{16}} = \sqrt[8]{(x^4y)^8} - xy^4 =$$

$$= x^4y - xy^4;$$

$$6) \left(\left(\sqrt[5]{\sqrt{a^5a}} \right)^5 - \sqrt[5]{a} \right) : \sqrt[10]{a^2} = \left(a\sqrt[5]{a} - \sqrt[5]{a} \right) : \sqrt[5]{a} = \frac{(a-1)\sqrt[5]{a}}{\sqrt[5]{a}} = a-1.$$

114.

$$1) \sqrt{7} \cdot \sqrt{14} : \sqrt{3} = \sqrt{\frac{98}{3}} \approx 5,72;$$

$$2) \sqrt{6,7} \cdot \sqrt{23} \cdot \sqrt{0,37} = \sqrt{6,7 \cdot 23 \cdot 0,37} = \sqrt{57,017} \approx 7,55;$$

$$3) \sqrt{(1,34)^{-7}} \cdot \sqrt{(0,43)^{-7}} = \sqrt{(1,34 \cdot 0,43)^{-7}} \approx 6,88;$$

$$4) \sqrt{(3,44)^{-9}} : \sqrt{(4,57)^{-9}} = \sqrt{(3,44 \cdot 4,57)^{-9}} \approx 3,59.$$

115.

$$1) \frac{\sqrt{3} \cdot \sqrt[3]{9}}{\sqrt[6]{3}} = \sqrt[6]{\frac{3^3 \cdot 3^4}{3}} = \sqrt[6]{3^6} = 3; 2) \frac{\sqrt[3]{7} \cdot \sqrt[4]{343}}{\sqrt[12]{7}} = \frac{7^{\frac{1}{3}} \cdot 7^{\frac{3}{4}}}{7^{\frac{1}{12}}} = 7^{\frac{13}{12} - \frac{1}{12}} = 7;$$

$$3) (\sqrt[3]{4} - \sqrt[3]{10} + \sqrt[3]{25})(\sqrt[3]{2} + \sqrt[3]{5}) = (\sqrt[3]{2})^3 + (\sqrt[3]{5})^3 = 2 + 5 = 7;$$

$$4) (\sqrt[3]{9} + \sqrt[3]{6} + \sqrt[3]{4})(\sqrt[3]{3} - \sqrt[3]{2}) = (\sqrt[3]{3})^3 - (\sqrt[3]{2})^3 = 3 - 2 = 1.$$

116.

$$\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}} = 2; (4+2\sqrt{3})(4-2\sqrt{3}) = 4;$$

$$\sqrt[2]{(4+2\sqrt{3})(4-2\sqrt{3})} = 2;$$

$$4+2\sqrt{3} - 2\sqrt{(4+2\sqrt{3})(4-2\sqrt{3})} + 4-2\sqrt{3} = 4;$$

$$\left(\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}\right)^2 = 2^2. \text{ Тогда } \sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}} = 2.$$

117.

$$1) \frac{\sqrt{a} - \sqrt{b}}{\sqrt[4]{a} - \sqrt[4]{b}} - \frac{\sqrt{a} + \sqrt[4]{ab}}{\sqrt[4]{a} + \sqrt[4]{b}} = \frac{(\sqrt[4]{a} - \sqrt[4]{b})(\sqrt[4]{a} + \sqrt[4]{b})}{\sqrt[4]{a} - \sqrt[4]{b}} - \frac{\sqrt[4]{a}(\sqrt[4]{a} + \sqrt[4]{b})}{\sqrt[4]{a} + \sqrt[4]{b}} = \\ = \sqrt[4]{a} + \sqrt[4]{b} - \sqrt[4]{a} = \sqrt[4]{b};$$

$$2) \frac{a-b}{\sqrt[3]{a} - \sqrt[3]{b}} + \frac{a+b}{\sqrt[3]{a} + \sqrt[3]{b}} = \frac{(\sqrt[3]{a} - \sqrt[3]{b})(\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2})}{\sqrt[3]{a} - \sqrt[3]{b}} + \\ + \frac{(\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{a^2})}{\sqrt[3]{a} + \sqrt[3]{b}} = \sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2} + \\ + \sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{a^2} = 2\sqrt[3]{a^2} + 2\sqrt[3]{b^2} = 2(\sqrt[3]{a^2} + \sqrt[3]{b^2});$$

$$3) \frac{1}{\sqrt[4]{a}-\sqrt[4]{b}} - \frac{1}{\sqrt[4]{a}+\sqrt[4]{b}} \cdot (\sqrt{a}-\sqrt{b}) = \left(\frac{\sqrt[4]{a}+\sqrt[4]{b}}{\sqrt{a}-\sqrt{b}} - \frac{\sqrt[4]{a}-\sqrt[4]{b}}{\sqrt{a}-\sqrt{b}} \right) (\sqrt{a}-\sqrt{b}) =$$

$$= \left(\frac{\sqrt[4]{a}+\sqrt[4]{b}}{\sqrt{a}-\sqrt{b}} - \frac{\sqrt[4]{a}-\sqrt[4]{b}}{\sqrt{a}-\sqrt{b}} \right) (\sqrt{a}-\sqrt{b}) = \sqrt[4]{a} - \sqrt[4]{a} + 2\sqrt[4]{b} = 2\sqrt[4]{b};$$

$$4) \left(\frac{a+b}{\sqrt[3]{a}+\sqrt[3]{b}} - \sqrt[3]{ab} \right) : (\sqrt[3]{a}-\sqrt[3]{b})^2 =$$

$$= \frac{(\sqrt[3]{a}+\sqrt[3]{b}) \left(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2} \right) - \sqrt[3]{ab} (\sqrt[3]{a} + \sqrt[3]{b})}{\sqrt[3]{a} + \sqrt[3]{b}} : (\sqrt[3]{a} - \sqrt[3]{b})^2 =$$

$$= \left(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2} - \sqrt[3]{ab} \right) : (\sqrt[3]{a} - \sqrt[3]{b})^2 = (\sqrt[3]{a} - \sqrt[3]{b})^2 : (\sqrt[3]{a} - \sqrt[3]{b})^2 =$$

118.

$$1) \sqrt{x^3} = x^{\frac{3}{2}};$$

$$2) \sqrt[3]{a^4} = a^{\frac{4}{3}};$$

$$3) \sqrt[4]{b^3} = b^{\frac{3}{4}};$$

$$4) \sqrt[5]{x^{-1}} = x^{-\frac{1}{5}};$$

$$5) \sqrt[6]{a} = a^{\frac{1}{6}};$$

$$6) \sqrt[7]{b^{-3}} = b^{-\frac{3}{7}}.$$

119.

$$1) x^{\frac{1}{4}} = \sqrt[4]{x};$$

$$2) y^{\frac{2}{5}} = \sqrt[5]{y^2};$$

$$3) a^{-\frac{5}{6}} = \sqrt[6]{a^{-5}};$$

$$4) b^{-\frac{1}{3}} = \sqrt[3]{b^{-1}};$$

$$5) (2x)^{\frac{1}{2}} = \sqrt{2x};$$

$$6) (3b)^{-\frac{2}{3}} = \sqrt[3]{(3b)^{-2}}.$$

120.

$$1) 64^{\frac{1}{2}} = \sqrt{64} = 8;$$

$$2) 27^{\frac{1}{3}} = \sqrt[3]{27} = 3;$$

$$3) 8^{\frac{2}{3}} = \sqrt[3]{64} = 4;$$

$$4) 81^{\frac{3}{4}} = \sqrt[4]{81^3} = 3^3 = 27;$$

$$5) 16^{-\frac{3}{4}} = \sqrt[4]{16^{-3}} = \frac{1}{2^3} = \frac{1}{8};$$

$$6) 9^{-\frac{3}{2}} = \sqrt{9^{-3}} = \frac{1}{3^3} = \frac{1}{27}.$$

121.

1) $2^{\frac{4}{5}} \cdot 2^{\frac{11}{5}} = 2^{\frac{15}{5}} = 2^3 = 8;$

2) $5^{\frac{2}{7}} \cdot 5^{\frac{5}{7}} = 5;$

3) $9^{\frac{2}{3}} : 9^{\frac{1}{6}} = 9^{\frac{3}{6}} = 9^{\frac{1}{2}} = 3;$

4) $4^{\frac{1}{3}} : 4^{\frac{5}{6}} = \frac{1}{2};$

5) $(7^{-3})^{\frac{2}{3}} = 7^2 = 49;$

6) $\left(8^{\frac{1}{12}}\right)^{-4} = 8^{-\frac{1}{3}} = \frac{1}{2}.$

122.

1) $9^{\frac{2}{5}} \cdot 27^{\frac{2}{5}} = 3^{\frac{4}{5}} \cdot 3^{\frac{6}{5}} = 3^{\frac{10}{5}} = 3^2 = 9;$

2) $7^{\frac{2}{3}} \cdot 49^{\frac{2}{3}} = 7^{\frac{2}{3}} \cdot 7^{\frac{4}{3}} = 7^{\frac{6}{3}} = 7^2 = 49;$

3) $144^{\frac{3}{4}} : 9^{\frac{3}{4}} = \left(\frac{144}{9}\right)^{\frac{3}{4}} = 16^{\frac{3}{4}} = 2^3 = 8;$

4) $150^{\frac{3}{2}} : 6^{\frac{3}{2}} = \left(\frac{150}{6}\right)^{\frac{3}{2}} = 25^{\frac{3}{2}} = 5^3 = 125.$

123.

1) $\left(\frac{1}{16}\right)^{\frac{3}{4}} + \left(\frac{1}{8}\right)^{\frac{4}{3}} = 2^3 + 2^4 = 8 + 16 = 24;$

2) $(0,04)^{\frac{3}{2}} - (0,125)^{\frac{2}{3}} = \left(\frac{1}{25}\right)^{\frac{3}{2}} - \left(\frac{1}{8}\right)^{\frac{2}{3}} = 25^{\frac{3}{2}} - 8^{\frac{2}{3}} = 5^3 - 2^2 =$
 $= 125 - 4 = 121;$

3) $8^{\frac{9}{7}} : 8^{\frac{2}{7}} - 3^{\frac{6}{5}} \cdot 3^{\frac{4}{5}} = 8 - 3^2 = 8 - 9 = -1;$

4) $(5^{-\frac{2}{5}})^{-5} + ((0,2)_4^3)^{-4} = 5^2 + \left(\frac{1}{5}\right)^{-3} = 25 + 125 = 150.$

124.

1) $\sqrt[3]{a} \cdot \sqrt[6]{a} = \sqrt[6]{a^2} \cdot \sqrt[6]{a} = \sqrt[6]{a^3} = \sqrt{a}$, при $a=0,09$, $\sqrt{a} = \sqrt{0,09} = 0,3;$

2) $\sqrt{b} : \sqrt[6]{b} = \sqrt[6]{b^3} : \sqrt[6]{b} = \sqrt[6]{b^2} = \sqrt[3]{b}$, при $b = 27$, $\sqrt{b} = \sqrt[3]{27} = 3;$

$$3) \frac{\sqrt{b} \cdot \sqrt[3]{b^2}}{\sqrt[6]{b}} = \frac{\sqrt[6]{b^3} \cdot \sqrt[6]{b^4}}{\sqrt[6]{b}} = \sqrt[6]{b^6} = b = 1,3;$$

$$4) \sqrt[3]{a} \cdot \sqrt[4]{a} \cdot \sqrt[12]{a^5} = \sqrt[12]{a^4} \cdot \sqrt[12]{a^3} \cdot \sqrt[12]{a^5} = \sqrt[12]{a^{12}} = a = 2,7.$$

125.

$$1) a^{\frac{1}{3}} \cdot \sqrt{a} = a^{\frac{1}{3} + \frac{1}{2}} = a^{\frac{5}{6}};$$

$$2) b^{\frac{1}{2}} \cdot b^{\frac{1}{3}} \cdot \sqrt[6]{b} = b^{\frac{5}{6} + \frac{1}{6}} = b^{\frac{6}{6}} = b;$$

$$3) \sqrt[3]{b} : b^{\frac{1}{6}} = b^{\frac{1}{3} - \frac{1}{6}} = b^{\frac{2}{6} - \frac{1}{6}} = b^{\frac{1}{6}};$$

$$4) a^{\frac{4}{3}} : \sqrt[3]{a} = a^{\frac{4}{3} - \frac{1}{3}} = a;$$

$$5) x^{1,7} \cdot x^{2,8} : \sqrt{x^5} = x^{4,5} : x^{2,5} = x^{4,5-2,5} = x^2;$$

$$6) y^{-3,8} : y^{-2,3} \cdot \sqrt{y^3} = y^{-3,8+2,3+\frac{3}{2}} = y^0 = 1.$$

126.

$$1) 2^{2-3\sqrt{5}} \cdot 8^{\sqrt{5}} = 2^{2-3\sqrt{5}+3\sqrt{5}} = 2^2 = 4;$$

$$2) 3^{1+2\sqrt[3]{2}} : 9^{\sqrt[3]{2}} = 3^{1+2\sqrt[3]{2}-2\sqrt[3]{2}} = 3;$$

$$3) 6^{1+2\sqrt{3}} : \left(4^{\sqrt{3}} \cdot 9^{\sqrt{3}}\right) = 6^{1+2\sqrt{3}} : 6^{2\sqrt{3}} = 6^{1+2\sqrt{3}-2\sqrt{3}} = 6;$$

$$4) \left(5^{1+\sqrt{2}}\right)^{1-\sqrt{2}} = 5^{1-2} = 5^{-1} = \frac{1}{5}.$$

127.

$$1) (a^4)^{\frac{3}{4}} \cdot (b^{\frac{2}{3}})^{-6} = a^{-3} \cdot b^4;$$

$$2) \left(\left(\frac{a^6}{b^{-3}} \right)^4 \right)^{\frac{1}{12}} = (a^{24} b^{12})^{\frac{1}{12}} = a^2 b;$$

$$3) \left(\sqrt{x^{0,4} \cdot y^{1,2}} \right)^{10} = (x^{0,2} \cdot y^{0,6})^{10} = x^2 \cdot y^6;$$

$$4) x^{-2\sqrt{2}} \cdot \left(\frac{1}{x^{-\sqrt{2}-1}} \right)^{\sqrt{2}+1} = x^{-2\sqrt{2}} \cdot x^{2+2\sqrt{2}+1} = x^{-2\sqrt{2}+3+2\sqrt{2}} = x^3.$$

128.

$$1) \frac{a^{\frac{4}{3}} \left(a^{-\frac{1}{3}} + a^{\frac{2}{3}} \right)}{a^{\frac{1}{4}} \left(a^{\frac{3}{4}} + a^{-\frac{1}{4}} \right)} = \frac{a^{\frac{4}{3} - \frac{1}{3}} + a^{\frac{4}{3} + \frac{2}{3}}}{a^{\frac{1}{4} + \frac{3}{4}} + a^{\frac{1}{4} - \frac{1}{4}}} = \frac{a + a^2}{a + 1} = \frac{a(a+1)}{a+1} = a;$$

$$2) \frac{b^{\frac{1}{5}} \cdot \left(\sqrt[5]{b^4} - \sqrt[5]{b^{-1}} \right)}{b^{\frac{2}{3}} \left(\sqrt[3]{b} - \sqrt[3]{b^{-2}} \right)} = \frac{b^{\frac{1}{5} + \frac{4}{5}} - b^{\frac{1}{5} - \frac{1}{5}}}{b^{\frac{2}{3} + \frac{1}{3}} - b^{\frac{2}{3} - \frac{2}{3}}} = \frac{b - 1}{b - 1} = 1;$$

$$3) \frac{a^{\frac{5}{3}} \cdot b^{-1} - ab^{-\frac{1}{3}}}{\sqrt[3]{a^2} - \sqrt[3]{b^2}} = \frac{ab^{-1} \left(a^{\frac{2}{3}} - b^{\frac{2}{3}} \right)}{a^{\frac{2}{3}} - b^{\frac{2}{3}}} = \frac{a}{b};$$

$$4) \frac{a^{\frac{1}{3}} \sqrt[6]{b} + b^{\frac{1}{3}} \sqrt[6]{a}}{\sqrt[6]{a} + \sqrt[6]{b}} = \frac{a^{\frac{1}{3}} b^{\frac{1}{6}} \left(b^{\frac{1}{6} - \frac{1}{3}} + a^{\frac{1}{6} - \frac{1}{3}} \right)}{a^{\frac{1}{6}} + b^{\frac{1}{6}}} = \frac{a^{\frac{1}{3}} b^{\frac{1}{6}} \left(b^{-\frac{1}{6}} + a^{-\frac{1}{6}} \right)}{a^{\frac{1}{6}} + b^{\frac{1}{6}}} = a^{\frac{1}{3}} b^{\frac{1}{6}}.$$

129.

$$1) \left(2^{\frac{5}{3}} \cdot 3^{-\frac{1}{3}} - 3^{\frac{5}{3}} \cdot 2^{-\frac{1}{3}} \right) \cdot \sqrt[3]{6} = 2^{\frac{1}{3}} \cdot 3^{-\frac{1}{3}} (2^2 - 3^2) \cdot \sqrt[3]{2} \cdot \sqrt[3]{3} = \\ = \frac{4-9}{\sqrt[3]{2} \cdot \sqrt[3]{3}} \cdot \sqrt[3]{2} \cdot \sqrt[3]{3} = -5;$$

$$2) \left(5^{\frac{1}{4}} \cdot 2^{\frac{3}{4}} - 2^{\frac{1}{4}} \cdot 5^{\frac{3}{4}} \right) \cdot \sqrt[4]{1000} = \left(\frac{5^4}{2^4} - \frac{2^4}{5^4} \right) \cdot \sqrt[4]{10^3} = \frac{5-2}{10^4} \cdot 10^{\frac{3}{4}} = 3;$$

$$3) \left(2^{\sqrt{2}} \right)^{\sqrt{2}} + \left(3^{\sqrt{3}+1} \right)^{\left(\sqrt{3}-1 \right)} = 2^2 + 3^{3-1} = 2^2 + 3^2 = 4 + 9 = 13;$$

$$4) \left((0,5)^{\frac{3}{5}} \right)^{-5} - (4^{-0,3})^{\frac{5}{3}} = \left(\frac{1}{2} \right)^{-3} - 4^{\frac{1}{2}} = 8 - 2 = 6.$$

130.

$$1) a^{\frac{1}{9}} \cdot \sqrt[6]{a^3 \sqrt{a}} = a^{\frac{1}{9}} \left(a a^{\frac{1}{3}} \right)^{\frac{1}{6}} = a^{\frac{1}{9}} \left(a^{\frac{4}{3}} \right)^{\frac{1}{6}} = a^{\frac{1}{9} + \frac{2}{9}} = a^{\frac{1}{3}};$$

$$2) \left(\sqrt[3]{ab^{-2}} + (ab)^{-\frac{1}{6}} \right) \cdot \sqrt[6]{ab^4} = \left(a^{\frac{1}{3}b^{-\frac{2}{3}} + a^{-\frac{1}{6}b^{-\frac{1}{6}}} \right) a^{\frac{1}{6}} \cdot b^{\frac{2}{3}} =$$
$$= a^{\frac{1}{2}} + b^{\frac{1}{2}} = \sqrt{a} + \sqrt{b};$$

$$3) b^{\frac{1}{12}} \cdot \sqrt[3]{b^4 \sqrt{b}} = b^{\frac{1}{12}} \left(b b^{\frac{1}{4}} \right)^{\frac{1}{3}} = b^{\frac{1}{12}} \left(b^{\frac{5}{4}} \right)^{\frac{1}{3}} = b^{\frac{1}{12}} \cdot b^{\frac{5}{12}} = b^{\frac{1}{2}} = \sqrt{b};$$

$$4) \left(\sqrt[3]{a} + \sqrt[3]{b} \right) \left(a^{\frac{2}{3}} + b^{\frac{2}{3}} - \sqrt[3]{ab} \right) = \left(\sqrt[3]{a} + \sqrt[3]{b} \right) \left((\sqrt[3]{a})^2 + (\sqrt[3]{b})^2 - \sqrt[3]{a} \sqrt[3]{b} \right) =$$
$$= (\sqrt[3]{a})^3 + (\sqrt[3]{b})^3 = a + b.$$

131.

$$1) \frac{x-y}{x^{\frac{1}{2}} + y^{\frac{1}{2}}} = \frac{\left(x^{\frac{1}{2}} + y^{\frac{1}{2}} \right) \left(x^{\frac{1}{2}} - y^{\frac{1}{2}} \right)}{x^{\frac{1}{2}} + y^{\frac{1}{2}}} = x^{\frac{1}{2}} - y^{\frac{1}{2}};$$

$$2) \frac{\sqrt{a} - \sqrt{b}}{a^{\frac{1}{4}} - b^{\frac{1}{4}}} = \frac{\left(a^{\frac{1}{4}} - b^{\frac{1}{4}} \right) \left(a^{\frac{1}{4}} + b^{\frac{1}{4}} \right)}{a^{\frac{1}{4}} - b^{\frac{1}{4}}} = a^{\frac{1}{4}} + b^{\frac{1}{4}};$$

$$3) \frac{m^{\frac{1}{2}} + n^{\frac{1}{2}}}{m + 2\sqrt{mn} + n} = \frac{m^{\frac{1}{2}} + n^{\frac{1}{2}}}{\left(m^{\frac{1}{2}} + n^{\frac{1}{2}} \right)^2} = \frac{1}{m^{\frac{1}{2}} + n^{\frac{1}{2}}};$$

$$4) \frac{c - 2c^{\frac{1}{2}} + 1}{\sqrt{c} - 1} = \frac{\left(c^{\frac{1}{2}} - 1 \right)^2}{c^{\frac{1}{2}} - 1} = c^{\frac{1}{2}} - 1.$$

132.

$$1) \left(1 - 2\sqrt{\frac{b}{a}} + \frac{b}{a}\right) : \left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right)^2 = \left(1 - \sqrt{\frac{b}{a}}\right)^2 \cdot \frac{1}{(\sqrt{a} - \sqrt{b})^2} =$$

$$= \frac{(\sqrt{a} - \sqrt{b})^2}{a \cdot (\sqrt{a} - \sqrt{b})^2} = \frac{1}{a};$$

$$2) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right) : \left(2 + \sqrt[3]{\frac{a}{b}} + \sqrt[3]{\frac{b}{a}}\right) = (\sqrt[3]{a} + \sqrt[3]{b}) : \frac{\sqrt[3]{a^2} + 2\sqrt[3]{ab} + \sqrt[3]{b^2}}{\sqrt[3]{ab}} =$$

$$= \frac{\sqrt[3]{ab}(\sqrt[3]{a} + \sqrt[3]{b})}{(\sqrt[3]{a} + \sqrt[3]{b})^2} = \frac{\sqrt[3]{ab}}{\sqrt[3]{a} + \sqrt[3]{b}};$$

$$3) \frac{a^{\frac{1}{4}} - a^{\frac{9}{4}}}{a^4 - a^4} - \frac{b^{\frac{1}{2}} - b^{\frac{3}{2}}}{b^2 + b^{\frac{1}{2}}} = \frac{(1 - a^2)a^{\frac{1}{4}}}{(1 - a)a^4} - \frac{(1 - b^2)b^{\frac{1}{2}}}{(1 + b)b^{\frac{1}{2}}} =$$

$$= 1 + a - (1 - b) = a + b;$$

$$4) \frac{\sqrt{a} - a^{-\frac{1}{2}}b}{1 - \sqrt{a^{-1}}b} - \frac{\sqrt[3]{a^2} - a^{-\frac{1}{3}}b}{\sqrt[6]{a} + a^{-\frac{1}{3}}\sqrt{b}} = \frac{(a - b)a^{-\frac{1}{2}}}{\sqrt{a} - \sqrt{b}} - \frac{(a - b)a^{-\frac{1}{3}}}{\sqrt{a} + \sqrt{b}}$$

$$= \frac{a - b}{\sqrt{a} - \sqrt{b}} - \frac{a - b}{\sqrt{a} + \sqrt{b}} = \frac{(a - b)(\sqrt{a} + \sqrt{b})}{a - b} - \frac{(\sqrt{a} - \sqrt{b})(a - b)}{a - b} =$$

$$= \sqrt{a} + \sqrt{b} - \sqrt{a} + \sqrt{b} = 2\sqrt{b}.$$

133.

$$1) \frac{a^{\frac{3}{2}}}{\sqrt{a} + \sqrt{b}} - \frac{ab^{\frac{1}{2}}}{\sqrt{b} - \sqrt{a}} - \frac{2a^2 - 4ab}{a - b} = \frac{a^{\frac{3}{2}}}{\sqrt{a} + \sqrt{b}} + \frac{b^{\frac{1}{2}}a}{\sqrt{a} - \sqrt{b}} -$$

$$- \frac{2a^2 - 4ab}{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})} = \frac{a^{\frac{3}{2}}(\sqrt{a} - \sqrt{b}) + ab^{\frac{1}{2}}(\sqrt{a} + \sqrt{b}) - 2a^2 + 4ab}{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})} =$$

$$= \frac{a^2 - a^{\frac{3}{2}}b^{\frac{1}{2}} + a^{\frac{3}{2}}\sqrt{b} + ab - 2a^2 + 4ab}{a - b} = \frac{5ab - a^2}{a - b};$$

$$\begin{aligned}
2) \quad & \frac{3xy - y^2}{x - y} - \frac{y\sqrt{y}}{\sqrt{x} - \sqrt{y}} - \frac{y\sqrt{x}}{\sqrt{x} + \sqrt{y}} = \\
& = \frac{3xy - y^2}{(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})} - \frac{y\sqrt{y}}{\sqrt{x} - \sqrt{y}} - \\
& - \frac{y\sqrt{x}}{\sqrt{x} + \sqrt{y}} = \frac{3xy - y^2 - y\sqrt{y}(\sqrt{x} + \sqrt{y}) - y\sqrt{x}(\sqrt{x} - \sqrt{y})}{x - y} = \\
& = \frac{3xy - y^2 - y^{\frac{3}{2}}\sqrt{x} - y^2 - yx + y^{\frac{3}{2}}\sqrt{x}}{x - y} = \frac{2xy - 2y^2}{x - y} = \frac{2y(x - y)}{x - y} = 2y; \\
3) \quad & \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}} - \frac{\sqrt[3]{a} + \sqrt[3]{b}}{a^{\frac{2}{3}} - \sqrt[3]{ab} + b^{\frac{2}{3}}} = \frac{a^{\frac{2}{3}} - \sqrt[3]{ab} + b^{\frac{2}{3}} - (\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a} + \sqrt[3]{b})}{a + b} = \\
& = \frac{\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2} - \sqrt[3]{a^2} - 2\sqrt[3]{ab} - \sqrt[3]{b^2}}{a + b} = \frac{-3\sqrt[3]{ab}}{a + b}; \\
4) \quad & \frac{\sqrt[3]{a^2} - \sqrt[3]{b^2}}{\sqrt[3]{a} - \sqrt[3]{b}} - \frac{a - b}{a^{\frac{2}{3}} + \sqrt[3]{ab} + b^{\frac{2}{3}}} = \frac{(\sqrt[3]{a} - \sqrt[3]{b})(\sqrt[3]{a} - \sqrt[3]{b})}{\sqrt[3]{a} - \sqrt[3]{b}} - \\
& - \frac{(\sqrt[3]{a} - \sqrt[3]{b}) \cdot (\sqrt[3]{a} + \sqrt[3]{ab} + \sqrt[3]{b^2})}{\sqrt[3]{a} + \sqrt[3]{ab} + \sqrt[3]{b^2}} = \sqrt[3]{a} + \sqrt[3]{b} - \sqrt[3]{a} + \sqrt[3]{b} = 2\sqrt[3]{b}.
\end{aligned}$$

134.

$$\begin{aligned}
1) \quad & \frac{(a - b)}{\sqrt[3]{a} - \sqrt[3]{b}} - \frac{a + b}{a^{\frac{1}{3}} + b^{\frac{1}{3}}} = \frac{(\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2})(\sqrt[3]{a} - \sqrt[3]{b})}{\sqrt[3]{a} - \sqrt[3]{b}} - \\
& - \frac{(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2})(\sqrt[3]{a} + \sqrt[3]{b})}{\sqrt[3]{a} + \sqrt[3]{b}} = \\
& = \sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2} - \sqrt[3]{a^2} + \sqrt[3]{ab} - \sqrt[3]{b^2} = 2\sqrt[3]{ab};
\end{aligned}$$

$$2) \frac{a+b}{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}} - \frac{a-b}{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}} = \frac{\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}} \right) \left(a^{\frac{1}{3}} + b^{\frac{1}{3}} \right)}{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}} - \frac{\left(a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}} \right) \left(a^{\frac{1}{3}} - b^{\frac{1}{3}} \right)}{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}} = a^{\frac{1}{3}} + b^{\frac{1}{3}} - \left(a^{\frac{1}{3}} - b^{\frac{1}{3}} \right) = 2b^{\frac{1}{3}};$$

$$3) \frac{a^{\frac{2}{3}} + b^{\frac{2}{3}}}{a-b} - \frac{1}{a^{\frac{1}{3}} - b^{\frac{1}{3}}} = \frac{a^{\frac{2}{3}} + b^{\frac{2}{3}} - \left(a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}} \right)}{a-b} = \frac{\sqrt[3]{ab}}{b-a};$$

$$4) \frac{a^{\frac{1}{3}} - b^{\frac{1}{3}}}{a+b} + \frac{1}{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}} = \frac{a^{\frac{1}{3}} - b^{\frac{1}{3}} + a^{\frac{1}{3}} + b^{\frac{1}{3}}}{a+b} = \frac{2\sqrt[3]{a}}{a+b}.$$

135.

$$1) \sqrt[3]{3} + \sqrt[3]{4} \approx 3,02; \quad 2) \sqrt[3]{7} + \sqrt[3]{10} \approx 2,04; \quad 3) 5^{\sqrt{3}} \approx 16,24;$$

$$4) (\sqrt[3]{2})^{\sqrt{3}} \approx 1,49; \quad 5) \pi^{\pi} \approx 36,46.$$

136.

$$1) 2^{\frac{1}{3}} < 3^{\frac{1}{3}}; \quad 2) 5^{-\frac{4}{5}} < 3^{-\frac{4}{5}}, \quad \text{т.к. } \frac{1}{\sqrt[5]{5^4}} < \frac{1}{\sqrt[5]{3^4}};$$

$$3) 5^{\sqrt{3}} < 7^{\sqrt{3}}; \quad 4) 21^{-\sqrt{2}} > 31^{-\sqrt{2}}, \quad \text{т.к. } \frac{1}{21^{\sqrt{2}}} > \frac{1}{31^{\sqrt{2}}}.$$

137.

$$1) (0,88)^{\frac{1}{6}} > \left(\frac{6}{11} \right)^{\frac{1}{6}}, \quad \text{т.к. } \frac{88}{100} > \frac{6}{11}, \text{ и } \left(\frac{88}{100} \right)^{\frac{1}{6}} > \left(\frac{6}{11} \right)^{\frac{1}{6}};$$

$$2) \left(\frac{5}{12} \right)^{\frac{1}{4}} < (0,41)^{\frac{1}{4}}, \quad \text{т.к. } \frac{12}{5} < \frac{100}{41} \text{ и } \left(\frac{12}{5} \right)^{\frac{1}{4}} < \left(\frac{100}{41} \right)^{\frac{1}{4}};$$

$$3) (4,09)^{\sqrt[3]{2}} < \left(4\frac{3}{25}\right)^{\sqrt[3]{2}}, \text{ т.к. } \left(4,09 < 4\frac{3}{25}\right);$$

$$4) \left(\frac{11}{12}\right)^{-\sqrt{5}} > \left(\frac{12}{13}\right)^{-\sqrt{5}}, \text{ т.к. } \frac{12}{11} > \frac{13}{12} \text{ и } \left(\frac{12}{11}\right)^{\sqrt{5}} > \left(\frac{13}{12}\right)^{\sqrt{5}}.$$

138.

$$1) 6^{2x} = 6^{\frac{1}{5}}.$$

$$2) 3^x = 27;$$

$$\text{Тогда } 2x = \frac{1}{5}.$$

$$3^x = 3^3;$$

$$\text{Отсюда } x = \frac{1}{10}.$$

$$x = 3.$$

$$3) 7^{1-3x} = 7^{10}.$$

$$4) 2^{2x+1} = 32,$$

$$\text{Поэтому } 1 - 3x = 10,$$

$$2^{2x+1} = 2^5.$$

$$x = -3.$$

$$\text{Тогда } 2x + 1 = 5, x = 2.$$

$$5) 4^{2+x} = 1;$$

$$6) \left(\frac{1}{5}\right)^{4x-3} = 5,$$

$$4^{2+x} = 4^0.$$

$$5^{3-4x} = 5,$$

$$\text{Поэтому } 2 + x = 0,$$

$$3 - 4x = 1,$$

$$x = -2.$$

$$x = \frac{1}{2}.$$

139.

$$1) \sqrt[7]{\left(\frac{1}{2} - \frac{1}{3}\right)^2} = \sqrt[7]{\left(\frac{3-2}{6}\right)^2} = \left(\frac{1}{6}\right)^{\frac{2}{7}};$$

$$\sqrt[7]{\left(\frac{1}{3} - \frac{1}{4}\right)^2} = \sqrt[7]{\left(\frac{4-3}{12}\right)^2} = \left(\frac{1}{12}\right)^{\frac{2}{7}}$$

$$\text{т.к. } \frac{1}{6} > \frac{1}{12}, \text{ а } \frac{2}{7} > 0,$$

$$\text{то } \sqrt[7]{\left(\frac{1}{2} - \frac{1}{3}\right)^2} > \sqrt[7]{\left(\frac{1}{3} - \frac{1}{4}\right)^2}.$$

$$2) \sqrt[5]{\left(1\frac{1}{4}-1\frac{1}{5}\right)^3} \quad \text{и} \quad \sqrt[5]{\left(1\frac{1}{6}-1\frac{1}{7}\right)^3};$$

$$\sqrt[5]{\left(1\frac{1}{4}-1\frac{1}{5}\right)^3} = \sqrt[5]{\left(\frac{25-24}{20}\right)^3} = \left(\frac{1}{20}\right)^{\frac{3}{5}};$$

$$\sqrt[5]{\left(1\frac{1}{6}-1\frac{1}{7}\right)^3} = \sqrt[5]{\left(\frac{49-48}{42}\right)^3} = \left(\frac{1}{42}\right)^{\frac{3}{5}};$$

т.к. $\frac{1}{20} > \frac{1}{42}$, а $\frac{3}{5} > 0$,

то $\sqrt[5]{\left(1\frac{1}{4}-1\frac{1}{5}\right)^3} > \sqrt[5]{\left(1\frac{1}{6}-1\frac{1}{7}\right)^3}$.

140.

1) $3^{2-y} = 27$, $3^{2-y} = 3^3$. Тогда $2 - y = 3$ и $y = -1$.

2) $3^{5-2x} = 1$; $3^{5-2x} = 3^0$. Поэтому $5 - 2x = 0$ и $x = 2,5$.

3) $9^{\frac{1}{2}x-1} - 3 = 0$; $9^{\frac{1}{2}x-1} = 3$; $3^{2\left(\frac{1}{2}x-1\right)} = 3$. Тогда $x - 2 = 1$ и $x = 3$.

4) $27^{3-\frac{1}{3}y} - 81 = 0$; $3^{3\left(3-\frac{1}{3}y\right)} = 3^4$. Тогда $9 - y = 4$ и $y = 5$.

141.

1) $\left(\frac{1}{9}\right)^{2x-5} = 3^{5x-8}$; $\left(3^{-2}\right)^{2x-5} = 3^{5x-8}$;

$$3^{-4x+10} = 3^{5x-8}$$

Тогда $10 - 4x = 5x - 8$,

$9x = 18$ и $x = 2$.

2) $2^{4x-9} = \left(\frac{1}{2}\right)^{x-4}$; $2^{4x-9} = 2^{-x+4}$.

Поэтому $4x - 9 = -x + 4$,

$5x = 13$ и $x = 2,6$.

3) $8^x \cdot 4^{x+13} = \frac{1}{16}$;

$$2^{3x} \cdot 2^{2x+26} = 2^{-4}$$

Тогда $3x + 2x + 26 = -4$, $5x = -30$; $x = -6$.

$$4) \frac{25^{x-2}}{\sqrt{5}} = \left(\frac{1}{5}\right)^{x-7,5};$$

$$5^{2x-4-\frac{1}{2}} = 5^{-x+7,5}.$$

Тогда $2x - 4,5 = -x + 7,5$,

$3x = 12$ и $x = 4$.

142.

$$1) \left(\frac{1}{\sqrt{3}}\right)^{2x+1} = (3\sqrt{3})^x,$$

$$\left(3^{\frac{1}{2}}\right)^{2x+1} = 3^{\frac{3}{2}x},$$

$$3^{-x-\frac{1}{2}} = 3^{\frac{3x}{2}}.$$

Тогда $-x - \frac{1}{2} = \frac{3}{2}x$,

$$-2,5x = 0,5$$

и $x = -\frac{1}{5}$.

$$3) 9^{3x+4} \cdot \sqrt{3} = \frac{27^{x-1}}{\sqrt{3}},$$

$$(3^2)^{3x+4} \cdot 3 = (3^3)^{x-1},$$

$$3^{6x+8+1} = 3^{3x-3}.$$

Тогда $6x + 9 = 3x - 3$,

$3x = -12$ и $x = -4$.

143.

$$1) \log_7 49 = \log_7 7^2 = 2;$$

$$3) \log_{\frac{1}{2}} 4 = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-2} = -2;$$

$$2) (\sqrt[3]{2})^{x-1} = \left(\frac{2}{\sqrt[3]{2}}\right)^{2x},$$

$$2^{\frac{x-1}{3}} = 2^{\frac{4x}{3}}.$$

Поэтому $\frac{x-1}{3} = \frac{4}{3}x$,

$$x - 1 = 4x,$$

$$3x = -1$$

и $x = -\frac{1}{3}$.

$$4) \frac{8}{(\sqrt{2})^x} = 4^{3x-2} \sqrt{2},$$

$$\frac{2^3}{2^{\frac{1}{2}x}} = 2^{2(3x-2)} \cdot 2^{\frac{1}{2}}.$$

Тогда $3 - \frac{1}{2}x = 2(3x-2) + \frac{1}{2}$,

$$6\frac{1}{2}x = 6\frac{1}{2}$$

и $x = 1$.

$$2) \log_2 64 = \log_2 2^6 = 6;$$

$$4) \log_3 \frac{1}{27} = \log_3 3^{-3} = -3.$$

144.

1) $\lg 23 \approx 1,4$; 2) $\lg 131 \approx 2,1$; 3) $40 \lg 2 \approx 12$; 4) $57 \lg 3 \approx 27,2$.

146.

$$1) 10^{2x-1} = 7, 2x-1 = \lg 7, x = \frac{1+\lg 7}{2}, x \approx 0,92;$$

$$2) 10^{1-3x} = 6, 1-3x = \lg 6,$$

$$x = \frac{1+\lg 6}{3}, x \approx 0,07.$$

146.

$$1) (0,175)^0 + (0,36)^{-2} - 1^{\frac{4}{3}} = 1 + \left(\frac{100}{36}\right)^2 - 1 = \left(\frac{25}{9}\right)^2 = \frac{625}{81};$$

$$2) 1^{-0,43} - (0,008)^{\frac{1}{3}} + (15,1)^0 = 1 - \left(\frac{1000}{8}\right)^{\frac{1}{3}} + 1 =$$
$$= 2 - \sqrt[3]{\frac{10^3}{2^3}} = 2 - \frac{10}{2} = -3;$$

$$3) \left(\frac{4}{5}\right)^{-2} - \left(\frac{1}{27}\right)^{\frac{1}{3}} + 4 \cdot 379^0 = \left(\frac{5}{4}\right)^2 - \sqrt[3]{\frac{1}{27}} + 4 = \frac{25}{16} - \frac{1}{3} + 4 =$$
$$= \frac{25}{16} + \frac{11}{3} = \frac{251}{48} = 5\frac{11}{48};$$

$$4) (0,125)^{-\frac{1}{3}} + \left(\frac{3}{4}\right)^2 - (1,85)^0 = \frac{1}{\sqrt[3]{0,125}} + \frac{9}{16} - 1 = \frac{1}{0,5} + \frac{9}{16} - 1 =$$
$$= \frac{9}{16} + 2 - 1 = 1\frac{9}{16}.$$

147.

$$1) 9,3 \cdot 10^{-6} : (3,1 \cdot 10^{-5}) = \frac{9,3 \cdot 10^{-6}}{3,1 \cdot 10^{-5}} = 3 \cdot 10^{-1} = 0,3;$$

$$2) 1,7 \cdot 10^{-6} \cdot 3 \cdot 10^7 = 5,1 \cdot 10 = 51;$$

$$3) 8,1 \cdot 10^{16} \cdot 2 \cdot 10^{-14} = 16,2 \cdot 10^2 = 1620;$$

$$4) 6,4 \cdot 10^5 : (1,6 \cdot 10^7) = \frac{6,4 \cdot 10^5}{1,6 \cdot 10^7} = \frac{4}{10^2} = 0,04;$$

$$5) 2 \cdot 10^{-1} + \left(6^0 - \frac{1}{6}\right)^{-1} \cdot \left(\frac{1}{3}\right)^{-2} \cdot \left(\frac{1}{3}\right)^3 \cdot \left(-\frac{1}{4}\right)^{-1} = \frac{1}{5} + \frac{6}{5} \cdot \frac{3^2}{3^3} \cdot (-4) = \\ = \frac{1}{5} + \frac{2 \cdot (-4) \cdot 3}{5 \cdot 3} = \frac{1}{5} - \frac{8}{5} = -\frac{7}{5} = -1,4;$$

$$6) 3 \cdot 10^{-1} - \left(8^0 - \frac{1}{8}\right)^{-1} \cdot \left(\frac{1}{4}\right)^{-3} \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{5}{7}\right)^{-1} = \frac{3}{10} - \frac{8}{7} \cdot \frac{1}{4} \cdot \frac{7}{5} = \\ = \frac{3}{10} - \frac{2}{5} = \frac{3-4}{10} = -0,1.$$

148.

$$1) \left(\frac{x^{\frac{1}{3}} \cdot x^{\frac{5}{6}}}{x^{\frac{1}{6}}}\right)^{-2} = \left(\frac{x^{\frac{2}{6}} \cdot x^{\frac{5}{6}}}{x^{\frac{1}{6}}}\right)^{-2} = \left(\frac{x^{\frac{7}{6}}}{x^{\frac{1}{6}}}\right)^{-2} = x^{-2} = \frac{1}{x^2},$$

$$\text{при } x = \frac{7}{9} \cdot \frac{1}{x^2} = \frac{81}{49} = 1\frac{32}{49};$$

$$2) \left(\frac{a^{\frac{2}{3}} \cdot a^{\frac{1}{9}}}{a^{\frac{-2}{9}}}\right)^{-3} = \left(\frac{a^{\frac{6}{9}} \cdot a^{\frac{1}{9}}}{a^{\frac{-2}{9}}}\right)^{-3} = \left(a^{\frac{7}{9}} \cdot a^{\frac{2}{9}}\right)^{-3} = (a)^{-3} = \frac{1}{a^3},$$

$$\text{при } a = 0,1, a^3 = 0,001, \frac{1}{a^3} = 1000.$$

149.

$$1) \left(\sqrt[3]{125x} - \sqrt[3]{8x}\right) - \left(\sqrt[3]{27x} - \sqrt[3]{64x}\right) = \left(5\sqrt[3]{x} - 2\sqrt[3]{x}\right) - \left(3\sqrt[3]{x} - 4\sqrt[3]{x}\right) = 4\sqrt[3]{x};$$

$$2) \left(\sqrt[4]{x} + \sqrt[4]{16x}\right) + \left(\sqrt[4]{81x} - \sqrt[4]{625x}\right) = \\ = \sqrt[4]{x} + 2\sqrt[4]{x} + 3\sqrt[4]{x} - 5\sqrt[4]{x} = \sqrt[4]{x};$$

$$3) \left(\frac{3}{\sqrt{1+a}} + \sqrt{1-a}\right) : \frac{3 + \sqrt{1-a^2}}{\sqrt{1+a}} = \frac{(3 + \sqrt{1-a^2})\sqrt{1+a}}{\sqrt{1+a}(3 + \sqrt{1-a^2})} = 1;$$

$$4) \left(1 - \frac{x}{\sqrt{x^2 - y^2}}\right) : \left(\sqrt{x^2 - y^2} - x\right) = \frac{\sqrt{x^2 - y^2} - x}{\sqrt{x^2 - y^2}(\sqrt{x^2 - y^2} - x)} = \frac{1}{\sqrt{x^2 - y^2}}.$$

150.

1) $7^{5x-1} = 49$; $7^{5x-1} = 7^2$.

Тогда $5x - 1 = 2$; $5x = 3$ и $x = \frac{3}{5}$.

2) $(0,2)^{1-x} = 0,04$; $(0,2)^{1-x} = (0,2)^2$.

Поэтому $1 - x = 2$ и $x = -1$.

3) $\left(\frac{1}{7}\right)^{3x+3} = 7^{2x}$; $7^{-3x-3} = 7^{2x}$.

Значит, $-3x - 3 = 2x$; $-5x = 3$ и $x = -\frac{3}{5}$.

4) $3^{5x-7} = \left(\frac{1}{3}\right)^{2x}$; $3^{5x-7} = 3^{-2x}$.

Отсюда, $5x - 7 = -2x$; $7x = 7$ и $x = 1$.

Проверь себя

1.

1) $3^{-5} : 3^{-7} - 2^{-2} \cdot 2^4 + \left(\left(\frac{2}{3}\right)^{-1}\right)^3 = 3^2 - 2^2 + \frac{27}{8} = 9 - 4 + 3\frac{3}{8} = 8\frac{3}{8}$;

2) $\sqrt[5]{3^{10} \cdot 32} - \frac{\sqrt[3]{48}}{\sqrt[3]{2} \cdot \sqrt[3]{3}} = 3^2 \cdot 2 - \sqrt[3]{8} = 18 - 2 = 16$;

3) $25^{\frac{3}{2}} \cdot 25^{-1} + \left(5^3\right)^{\frac{2}{3}} : 5^3 - 48^{\frac{2}{3}} : 6^{\frac{2}{3}} = \sqrt{25} + 5^{-1} - 8^{\frac{2}{3}} =$
 $= 5 + \frac{1}{5} - 4 = 1,2$.

2.

$8600 = 8,6 \cdot 10^3$;

$0,0078 = 7,8 \cdot 10^{-3}$;

1) $8,6 \cdot 10^3 \cdot 7,8 \cdot 10^{-3} = 67,08$; 2) $8,6 \cdot 10^3 : 7,8 \cdot 10^{-3} = \frac{43}{39} \cdot 10^6$.

3.

1) $\frac{3x^{-9} \cdot 2x^5}{x^{-4}} = 6$; 2) $(x^{-1} + y^{-1}) \cdot \left(\frac{1}{xy}\right)^{-2} = \frac{y+x}{xy} \cdot (xy)^2 = (x+y)xy$.

4.

$$\frac{a^{\frac{5}{3}}}{\sqrt[3]{a^2 \cdot a^4}} = a^{\frac{5}{3}} \cdot a^{-\frac{2}{3}} \cdot a^{-\frac{3}{4}} = a \cdot a^{-\frac{3}{4}} = a^{1-\frac{3}{4}} = a^{\frac{1}{4}}, \text{ при } a = 81, \text{ то } a^{\frac{1}{4}} = 3.$$

5.

а) $(0,78)^{\frac{2}{3}} > (0,67)^{\frac{2}{3}}$, т.к. $0,78 > 0,67$, и показатель степени $\frac{2}{3} > 0$;

б) $(3,09)^{-\frac{1}{3}} < (3,08)^{-\frac{1}{3}}$, т.к. $3,09 > 3,08$, и показатель $-\frac{1}{3} < 0$.

151.

$$1) \left(\frac{1}{16}\right)^{-\frac{3}{4}} + 10000^{\frac{1}{4}} - \left(7\frac{19}{32}\right)^{\frac{1}{5}} = (16)^{\frac{3}{4}} + 10 - \left(\frac{243}{32}\right)^{\frac{1}{5}} = 2^3 + 10 - \frac{3}{2} = \\ = 8 + 10 - \frac{3}{2} = 16,5;$$

$$2) (0,001)^{-\frac{1}{3}} - 2^{-2} \cdot 64^{\frac{2}{3}} - 8^{-\frac{1}{3}} = 1000^{\frac{1}{3}} - \frac{1}{4} \cdot \sqrt[3]{64^2} - \left(\frac{1}{8}\right)^{\frac{4}{3}} = \\ = 10 - \frac{16}{4} - 3\sqrt{\left(\frac{1}{8}\right)^4} = 10 - 4 - \frac{1}{16} = 5\frac{15}{16};$$

$$3) 27^{\frac{2}{3}} - (-2)^{-2} + \left(3\frac{3}{8}\right)^{\frac{1}{3}} = \sqrt[3]{27^2} - \frac{1}{4} + \sqrt[3]{\frac{8}{27}} = 9 - \frac{1}{4} + \frac{2}{3} = 9\frac{5}{12};$$

$$4) (-0,5)^{-4} - 625 - \left(2\frac{1}{4}\right)^{-\frac{1}{2}} = 16 - 625 - \sqrt{\left(\frac{4}{9}\right)^3} = \\ = 16 - 625 - \frac{8}{27} = -609\frac{8}{27}.$$

152.

1) $\sqrt[4]{x^2 - 4}$ — имеет смысл, если выполнено $x^2 - 4 \geq 0$,
т.е. $(x-2)(x+2) \geq 0$.



Ответ: $x \in (-\infty; -2] \cup [2; +\infty)$.

2) $\sqrt[3]{x^2 - 5x + 6}$ – имеет смысл для любого x .

Ответ: $x \in (-\infty; +\infty)$.

3) $\sqrt[6]{\frac{x-2}{x+3}}$ – имеет смысл, если $\frac{x-2}{x+3} \geq 0$, при этом $x+3 \neq 0$

т.е. $x \neq -3$.



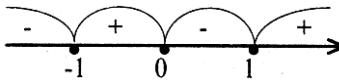
Ответ: $x \in (-\infty; -3) \cup [2; +\infty)$.

4) $\sqrt[4]{x^2 - 5x + 6}$ – имеет смысл, если $x^2 - 5x + 6 \geq 0$, тогда $(x-3)(x-2) \geq 0$.



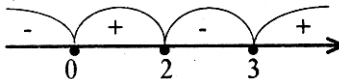
Ответ: $x \in (-\infty; +2] \cup [3; +\infty)$.

5) $\sqrt[8]{x^3 - x}$ – имеет смысл, если $x^3 - x \geq 0$, поэтому $x(x-1)(x+1) \geq 0$.



Ответ: $x \in [-1; 0] \cup [1; +\infty)$

6) $\sqrt[6]{x^3 - 5x^2 + 6x}$ – имеет смысл, если $x^3 - 5x^2 + 6x \geq 0$, тогда $x \cdot (x-3)(x-2) \geq 0$.



Ответ: $x \in [0; 2] \cup [3; +\infty)$.

153.

$$1) \frac{a^{\frac{1}{4}} - a^{-\frac{7}{4}}}{a^{\frac{1}{4}} - a^{-\frac{3}{4}}} = \frac{a^{-\frac{7}{4}}(a^2 - 1)}{a^{-\frac{3}{4}}(a-1)} = \frac{a^{-1}(a+1)(a-1)}{(a-1)} = \frac{a+1}{a} = 1 + \frac{1}{a};$$

$$2) \frac{a^{\frac{4}{3}} - a^{-\frac{2}{3}}}{a^{\frac{1}{3}} - a^{-\frac{2}{3}}} = \frac{a^{-\frac{2}{3}}(a^2 - 1)}{a^{-\frac{2}{3}}(a-1)} = \frac{(a+1)(a-1)}{(a-1)} = a+1;$$

$$3) \frac{b^{\frac{5}{4}} + 2b^{\frac{1}{4}} + b^{-\frac{3}{4}}}{b^{\frac{3}{4}} + b^{-\frac{1}{4}}} = \frac{b^{-\frac{3}{4}}(b^2 + 2b + 1)}{b^{-\frac{1}{4}}(b+1)} = \frac{(b+1)^2}{\sqrt{b}(b+1)} = \frac{b+1}{\sqrt{b}};$$

$$4) \frac{a^{-\frac{4}{3}}b^{-2} - a^{-2}b^{-\frac{4}{3}}}{a^{-\frac{5}{3}}b^{-2} - a^{-2}b^{-\frac{5}{3}}} = \frac{a^{-2}b^{-2}(a^{\frac{2}{3}} - b^{\frac{2}{3}})}{a^{-2}b^{-2}(a^{\frac{1}{3}} - b^{\frac{1}{3}})} = \frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}})(a^{\frac{1}{3}} - b^{\frac{1}{3}})}{a^{\frac{1}{3}} - b^{\frac{1}{3}}} =$$

$$= a^{\frac{1}{3}} + b^{\frac{1}{3}} = \sqrt[3]{a} + \sqrt[3]{b};$$

$$5) \frac{\sqrt{a^3b^{-1}} - \sqrt{a^{-1}b^3}}{\sqrt{ab^{-1}} - \sqrt{a^{-1}b}} = \frac{\frac{\sqrt{a^3}}{\sqrt{b}} - \frac{\sqrt{b^3}}{\sqrt{a}}}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{b}}{\sqrt{a}}} =$$

$$= \frac{\frac{\sqrt{a^4} - \sqrt{b^4}}{\sqrt{ab}}}{\frac{\sqrt{a^2} - \sqrt{b^2}}{\sqrt{ab}}} = \frac{\sqrt{a^4} - \sqrt{b^4}}{\sqrt{a^2} - \sqrt{b^2}} =$$

$$= \frac{a^2 - b^2}{a - b} = \frac{(a+b)(a-b)}{a-b} = a + b;$$

$$6) \frac{a^{\frac{3}{4}}b^{\frac{1}{4}} - a^{-\frac{1}{4}}b^{\frac{3}{4}}}{a^{\frac{1}{4}}b^{\frac{1}{4}} + a^{-\frac{1}{4}}b^{\frac{1}{4}}} = \frac{a^{-\frac{1}{4}}b^{\frac{1}{4}}(a - b)}{a^{-\frac{1}{4}}b^{\frac{1}{4}}\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)} = \frac{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})}{\sqrt{a} + \sqrt{b}} =$$

$$= \sqrt{a} - \sqrt{b};$$

$$7) \left(\frac{1 + \sqrt{ab}}{\sqrt[4]{ab}} + \frac{\sqrt[4]{a^3b} - \sqrt[4]{ab^3}}{\sqrt{b} - \sqrt{a}} \right)^{-2} \cdot \left(1 + \frac{b}{a} + 2\sqrt{\frac{b}{a}} \right)^{\frac{1}{2}} =$$

$$= \left(\frac{(1 + \sqrt{ab})(\sqrt{b} - \sqrt{a}) + \sqrt[4]{ab}(\sqrt[4]{a^3b} - \sqrt[4]{ab^3})}{\sqrt[4]{ab} \cdot (\sqrt{b} - \sqrt{a})} \right)^{-2} \cdot \left(\left(1 + \sqrt{\frac{b}{a}} \right)^2 \right)^{\frac{1}{2}} =$$

$$= \sqrt{ab} \cdot \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a}} = (\sqrt{a} + \sqrt{b}) \cdot \sqrt{b};$$

$$\begin{aligned}
& 8) \left(\frac{a+b}{\sqrt[3]{a^2} - \sqrt[3]{b^2}} + \frac{\sqrt[3]{ab^2} - \sqrt[3]{a^2b}}{\sqrt[3]{a^2} - 2\sqrt[3]{ab} + \sqrt[3]{b^2}} \right) : (\sqrt[6]{a} - \sqrt[6]{b}) = \\
& = \left(\frac{(\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2})}{(\sqrt[3]{a} - \sqrt[3]{b})(\sqrt[3]{a} + \sqrt[3]{b})} + \frac{\sqrt[3]{ab}(\sqrt[3]{b} - \sqrt[3]{a})}{(\sqrt[3]{a} - \sqrt[3]{b})^2} \right) : (\sqrt[6]{a} - \sqrt[6]{b}) = \\
& = \left(\frac{\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}}{\sqrt[3]{a} - \sqrt[3]{b}} - \frac{\sqrt[3]{ab}}{\sqrt[3]{a} - \sqrt[3]{b}} \right) : (\sqrt[6]{a} - \sqrt[6]{b}) = \\
& = \frac{\sqrt[3]{a^2} - 2\sqrt[3]{ab} + \sqrt[3]{b^2}}{\sqrt[3]{a} - \sqrt[3]{b}} : (\sqrt[6]{a} - \sqrt[6]{b}) = \frac{(\sqrt[3]{a} - \sqrt[3]{b})^2}{(\sqrt[3]{a} - \sqrt[3]{b})(\sqrt[6]{a} - \sqrt[6]{b})} = \\
& = \frac{\sqrt[3]{a} - \sqrt[3]{b}}{\sqrt[6]{a} - \sqrt[6]{b}} = \frac{(\sqrt[6]{a} + \sqrt[6]{b})(\sqrt[6]{a} - \sqrt[6]{b})}{\sqrt[6]{a} - \sqrt[6]{b}} = \sqrt[6]{a} + \sqrt[6]{b}.
\end{aligned}$$

154.

$$V_k = a^3;$$

$$V_{ш} = \frac{4}{3}\pi \cdot R^3,$$

если $V_k = V_{ш} = 100\text{см}^3$;

$$a = \sqrt[3]{V_k} = \sqrt[3]{10^2} \approx 4,64 \text{ см}; R = \sqrt[3]{\frac{V_{ш}}{\frac{4}{3}\pi}} = \sqrt[3]{\frac{3V_{ш}}{4\pi}} = \sqrt[3]{\frac{300}{4\pi}} \approx 2,88;$$

$2R = 5,74$, $2R > a$, следовательно, шар не поместится в куб, т.к. диаметр шара больше ребра куба.

155.

$$T = 2\pi \sqrt{\frac{l}{g}} \approx 2\pi \sqrt{\frac{0,185}{9,8}} \approx 2 \cdot 3,14 \cdot \sqrt{\frac{0,185}{9,8}} \approx 0,86\text{с}.$$

156.

а) $y(x) = x^2 - 4x + 5$,

$$y(-3) = (-3)(-3) - 4(-3) + 5 = 9 + 12 + 5 = 26,$$

$$y(-1) = (-1)(-1) - 4(-1) + 5 = 1 + 4 + 5 = 10,$$

$$y(0) = 0 - 0 + 5 = 5,$$

$$y(2) = 2^2 - 4 \cdot 2 + 5 = 4 - 8 + 5 = 1;$$

б) пусть $y(x) = 1$, значит $x^2 - 4x + 5 = 1$,

$$x^2 - 4x + 4 = 0; (x - 2)^2 = 0, \text{ тогда } x - 2 = 0, x = 2,$$

пусть $y(x) = 5$, значит $x^2 - 4x + 5 = 5; x^2 - 4x = 0$,

$$x(x - 4) = 0, \text{ тогда } x_1 = 4; x_2 = 0, \text{ если } y(x) = 10, \text{ то } x^2 - 4x + 5 = 10,$$

$$x^2 - 4x - 5 = 0, \text{ тогда } x_1 = 5, x_2 = -1, \text{ если } y(x) = 17, \text{ то } x^2 - 4x - 5 = 17,$$

$$x^2 - 4x - 12 = 0, \text{ тогда } x_1 = 6, x_2 = -2.$$

157.

$$y(x) = \frac{x+5}{x-1};$$

$$1) y(-2) = \frac{3}{-3} = -1, \quad y(0) = \frac{5}{-1} = -5,$$

$$y\left(\frac{1}{2}\right) = \frac{5,5}{-0,5} = -11, \quad y(3) = \frac{3+5}{3-1} = \frac{8}{2} = 4;$$

$$2) \text{ если } y(x) = -3, \text{ то } \frac{x+5}{x-1} = -3;$$

$$x + 5 + 3x - 3 = 0, \text{ при этом } x - 1 \neq 0,$$

$$\begin{cases} 4x = -2 \\ x \neq 1 \end{cases},$$

$$\text{тогда } x = -\frac{1}{2},$$

$$\text{если } y(x) = -2, \text{ то } \frac{x+5}{x-1} = -2,$$

$$x + 5 + 2x - 2 = 0, \text{ при этом } x - 1 \neq 0,$$

$$3x = -3, x \neq 1,$$

$$\text{значит, } x = -1,$$

$$\text{если } y(x) = 13, \text{ то } \frac{x+5}{x-1} = 13,$$

$$x + 5 - 13x + 13 = 0, \text{ при этом } x - 1 \neq 0,$$

$$-12x = -18, x \neq 1,$$

$$\text{значит, } x = 1,5,$$

$$\text{если } y(x) = 19, \text{ то } \frac{x+5}{x-1} = 19,$$

$$x + 5 - 19x + 19 = 0, \text{ при этом } x - 1 \neq 0,$$

$$-18x = -24, x \neq 1,$$

$$\text{поэтому, } x = \frac{4}{3}.$$

158.

$$1) y = 4x^2 - 5x + 1, x \in (-\infty; \infty);$$

$$2) y = 2 - x - x^2, x \in (-\infty; \infty);$$

$$3) y = \frac{2x-3}{x-3}, x \neq 3, x \in (-\infty; 3) \cup (3; +\infty);$$

$$4) y = \frac{3}{5-x^2}, x^2 \neq 5, x \in (-\infty; -\sqrt{5}) \cup (-\sqrt{5}; \sqrt{5}) \cup (\sqrt{5}; \infty);$$

$$5) y = \sqrt[4]{6-x}, 6-x \geq 0, x \in (-\infty; 6];$$

$$6) y = \sqrt{\frac{1}{x+7}}, x+7 > 0, x \in (-7; \infty).$$

159.

$$1) y = \frac{2x}{x^2 - 2x - 3}, x^2 - 2x - 3 \neq 0;$$

т.е. $(x-1)(x-3) \neq 0$; значит $x \neq 1, x \neq 3, x \in (-\infty; 1) \cup (1; 3) \cup (3; \infty)$;

$$2) y = \sqrt[6]{x^2 - 7x + 10},$$

тогда $x^2 - 7x + 10 \geq 0, (x-2)(x-5) \geq 0,$



$$x \in (-\infty; 2] \cup [5; +\infty);$$

$$3) y = \sqrt[8]{3x^2 - 2x + 5}, \text{ значит,}$$

$$3x^2 - 2x + 5 \geq 0.$$

Найдем корни уравнения

$$3x^2 - 2x + 5 = 0:$$

$$\frac{D}{4} = 1 - 15 = -14 < 0, \text{ корней нет, поэтому т.к. } 3 > 0 - \text{ ветви вверх,}$$

значит, $3x^2 - 2x + 5 > 0$, для любого $x, x \in (-\infty; \infty)$,

$$4) y = \sqrt[6]{\frac{2x+4}{3-x}}, \text{ тогда } \frac{2x+4}{3-x} \geq 0,$$

при этом $3-x \neq 0; x \neq 3; -2 \leq x < 3,$



$$x \in (-2; 3).$$

160.

$$y(x) = |2 - x| - 2;$$

$$1) y(-3) = |2 + 3| - 2 = 5 - 2 = 3,$$

$$y(-1) = |2 + 1| - 2 = 3 - 2 = 1,$$

$$y(1) = |2 - 1| - 2 = 1 - 2 = -1,$$

$$y(3) = |2 - 3| - 2 = 1 - 2 = -1,$$

2) если $y(x) = -2$, то

если $y(x) = 0$, то

если $y(x) = 2$, то

если $y(x) = 4$, то

$$|2 - x| - 2 = -2,$$

$$|2 - x| = 0 \text{ и } x = 2,$$

$$|2 - x| - 2 = 0,$$

$$|2 - x| = 2,$$

$$2 - x = 2 \text{ или } -2 + x = 2,$$

$$\text{тогда } x_1 = 4; x_2 = 0,$$

$$|2 - x| - 2 = 2,$$

$$|2 - x| = 4,$$

$$2 - x = 4 \text{ или } -2 + x = 4,$$

$$\text{значит } x_1 = -2; x_2 = 6,$$

$$|2 - x| - 2 = 4,$$

$$|2-x|=6,$$

$$2-x=6 \text{ или } -2+x=6,$$

$$\text{ПОЭТОМУ, } x_1=8; x_2=-4.$$

161.

$$1) y = \sqrt{\frac{x-2}{x+3}}, \text{ значит, } \frac{x-2}{x+3} \geq 0, x+3 \neq 0; x \neq -3;$$



$$x \in (-\infty; -3) \cup [2; \infty);$$

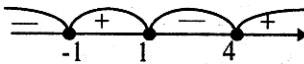
$$2) y = \sqrt[4]{(x-1)(x-2)(x-3)}; (x-1)(x-2)(x-3) \geq 0,$$



$$x \in [1; 2] \cup [3; +\infty);$$

$$3) y = \sqrt[3]{\frac{1-x}{1+x}}, \text{ тогда } 1+x \neq 0; x \neq -1, x \in (-\infty; -1) \cup (-1; \infty);$$

$$4) y = \sqrt{(x+1)(x-1)(x-4)}; (x+1)(x-1)(x-4) \geq 0$$



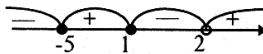
$$x \in [-1; 1] \cup [4; +\infty);$$

$$5) y = \sqrt[8]{\frac{x^2+4x-5}{x-2}},$$

$$\text{тогда } \frac{x^2+4x-5}{x-2} \geq 0,$$

$$x-2 \neq 0; x \neq 2$$

$$\frac{(x-1)(x+5)}{x-2} \geq 0, x \neq 2,$$



$$x \in [-5; 1] \cup (2; +\infty);$$

$$6) y = \sqrt[6]{x} + \sqrt{1+x}, \text{ тогда } \begin{cases} x \geq 0 \\ 1+x \geq 0 \end{cases} \begin{cases} x \geq 0 \\ x \geq -1 \end{cases}, x \geq 0,$$



$$x \in [0; +\infty).$$

162.

$$1) y = 3x^2 + 2x + 29.$$

Подставим координаты $M(-2; 1)$,

$$1 = 3 \cdot 4 - 4 + 29,$$

$1 \neq 37$, значит, не принадлежит;

$$2) y = |4 - 3x| - 9,$$

$M(-2; 1)$,

$$1 = |4 + 6| - 9,$$

$1 = 1$, значит, принадлежит;

$$3) y = \frac{x^2 + 3}{x - 1},$$

$$M(-2; 1); 1 = \frac{4 + 3}{-2 - 1}; 1 \neq -\frac{7}{3},$$

значит, не принадлежит;

$$4) y = |\sqrt{2 - x} - 5| - 2,$$

$$M(-2; 1), 1 = |\sqrt{2 - 2} - 5| - 2,$$

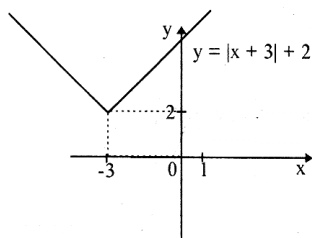
$$1 = |2 - 5| - 2, 1 = 3 - 2,$$

$1 = 1$, значит, принадлежит.

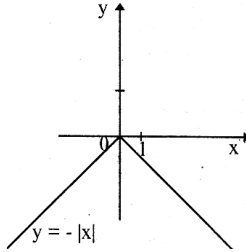
163.

$$1) y = |x + 3| + 2,$$

$$y = \begin{cases} x + 5, & x \geq -3 \\ -x - 1, & x < -3 \end{cases};$$



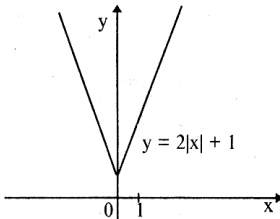
$$2) y = -|x|, y = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases};$$



$$3) y = 2|x| + 1,$$

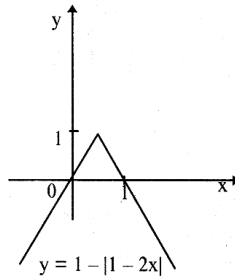
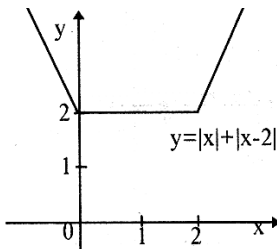
$$4) y = 1 - |1 - 2x|, y = \begin{cases} 2x, & x \leq \frac{1}{2} \\ -2x + 2, & x > \frac{1}{2} \end{cases};$$

$$y = \begin{cases} 2x + 1, & x \geq 0 \\ -2x + 1, & x < 0 \end{cases};$$



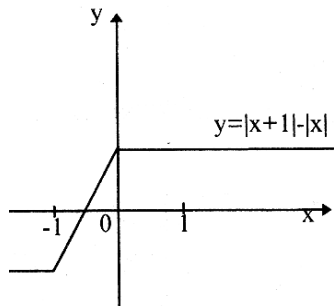
$$5) y = |x| + |x - 2|,$$

$$y = \begin{cases} -2x + 2, & x < 0 \\ 2, & 0 \leq x \leq 2 \\ 2x - 2, & x > 2 \end{cases};$$



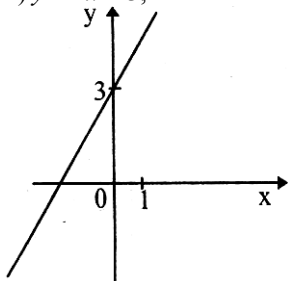
$$6) y = |x + 1| - |x|,$$

$$y = \begin{cases} -1, & x < -1 \\ 2x + 1, & -1 \leq x < 0 \\ 1, & x \geq 0 \end{cases}.$$



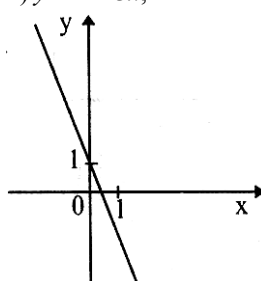
164.

1) $y = 2x + 3$,

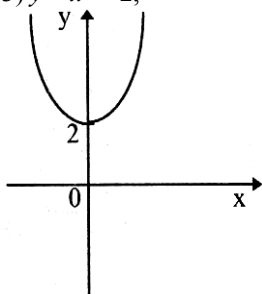


y возрастает, если $x \in (-\infty; +\infty)$; y убывает, если $x \in (-\infty; \infty)$;

2) $y = 1 - 3x$,

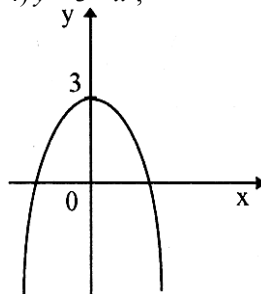


3) $y = x^2 + 2$,



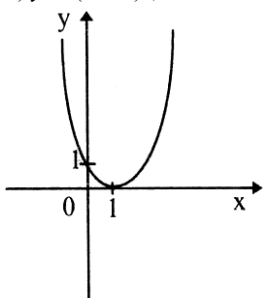
y возрастает, если $x \in (0; +\infty)$,
 y убывает, если $x \in (-\infty; 0)$;

4) $y = 3 - x^2$,



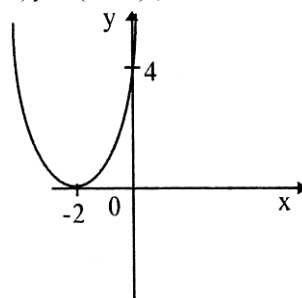
y возрастает, если $x \in (-\infty; 0)$,
 y убывает, если $x \in (0; +\infty)$;

5) $y = (1 - x)^2$,



y возрастает, если $x \in (1; +\infty)$,
 y убывает, если $x \in (-\infty; 1)$;

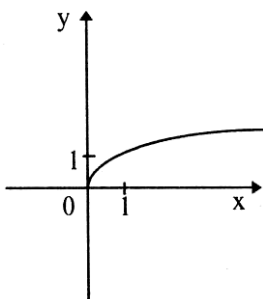
6) $y = (2 + x)^2$,



y возрастает, если $x \in (-2; +\infty)$,
 y убывает, если $x \in (-\infty; -2)$;

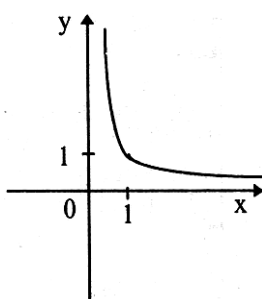
166.

$$1) y = x^{\frac{3}{7}}.$$



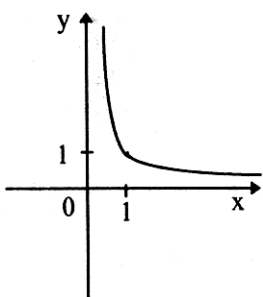
Ответ: возрастает.

$$2) y = x^{-\frac{3}{4}}.$$



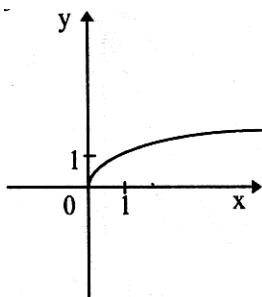
Ответ: убывает.

$$3) y = x^{-\sqrt{2}}.$$



Ответ: убывает.

$$4) y = x^{\sqrt{3}}.$$



Ответ: возрастает.

167.

$$1) x^{\frac{1}{2}} = 3;$$

$$2) x^{\frac{1}{4}} = 2;$$

$$3) x^{-\frac{1}{2}} = 3;$$

$$x = 3^2 = 9;$$

$$x = 2^4 = 16;$$

$$x = 3^{-2} = \frac{1}{9};$$

$$4) x^{-\frac{1}{4}} = 2;$$

$$5) x^{\frac{5}{6}} = 32;$$

$$6) x^{-\frac{4}{5}} = 81;$$

$$x = 2^{-4} = \frac{1}{16};$$

$$x = \sqrt[5]{32^6} = 2^6 = 64;$$

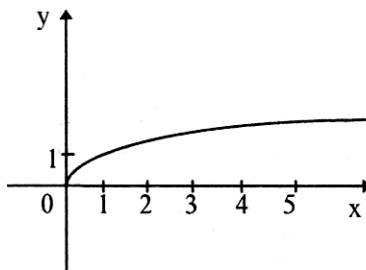
$$x = \sqrt[4]{81^{-5}} = \left(\frac{1}{3}\right)^5 = \frac{1}{243}.$$

168.

$$y = \sqrt[4]{x};$$

а) при $y = 0,5$; $x \approx 0,6$,

42



при $y = 1; x = 1$,
 при $y = 4; x = 256$,
 при $y = 2,5; x \approx 39$;

$$\text{б) } \sqrt[4]{1,5} \approx 1,2,$$

$$\sqrt[4]{2} \approx 1,3,$$

$$\sqrt[4]{2,5} \approx 1,4,$$

$$\sqrt[4]{3} \approx 1,5.$$

169.

$$1) \left\{ \begin{array}{l} y = x^{\frac{4}{3}}; \quad x^{\frac{4}{3}} = 625; \\ x = (625)^{\frac{3}{4}} = (5^4)^{\frac{3}{4}} = 5^3; 2) \\ y = 625; x = 125. \end{array} \right. \left\{ \begin{array}{l} y = x^{\frac{6}{5}}; \quad x^{\frac{6}{5}} = 64; \\ x = 64^{\frac{5}{6}} = (2^6)^{\frac{5}{6}} = 2^5; \\ y = 64; \quad x = 32. \end{array} \right.$$

Ответ: М (125, 625).

Ответ: М (32, 64).

$$3) \left\{ \begin{array}{l} y = x^{\frac{3}{2}}; \quad x^{\frac{3}{2}} = 216; \\ x = 216^{\frac{2}{3}} = (6^3)^{\frac{2}{3}} = 6^2; 4) \\ y = 216; \quad x = 36. \end{array} \right. \left\{ \begin{array}{l} y = x^{\frac{7}{3}}; \quad x^{\frac{7}{3}} = 128; \\ x = 128^{\frac{3}{7}} = (2^7)^{\frac{3}{7}} = 2^3; \\ y = 128; \quad x = 8. \end{array} \right.$$

Ответ: М (36, 216). Ответ: М (8, 128). 170.

$$1) y = x + \frac{1}{x}; \text{ пусть } x_1 < x_2, \quad y_1 = x_1 + \frac{1}{x_1} = \frac{x_1^2 + 1}{x_1};$$

$$y_2 = x_2 + \frac{1}{x_2} = \frac{x_2^2 + 1}{x_2};$$

$$\begin{aligned} y_1 - y_2 &= \frac{x_1^2 + 1}{x_1} - \frac{x_2^2 + 1}{x_2} = \frac{x_1^2 \cdot x_2 + x_2 - x_2^2 \cdot x_1 - x_1}{x_1 \cdot x_2} = \\ &= \frac{x_1 \cdot x_2 (x_1 - x_2) - (x_1 - x_2)}{x_1 \cdot x_2} = \frac{(x_1 - x_2) \cdot (x_1 \cdot x_2 - 1)}{x_1 \cdot x_2}, \end{aligned}$$

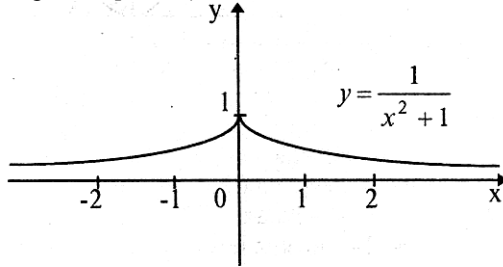
при $x_1, x_2 > 0$, но $x_1, x_2 < 1$, имеем $x_1 - x_2 < 0$, $x_1 \cdot x_2 > 0$, $x_1 \cdot x_2 - 1 < 0$

тогда $\frac{(x_1 - x_2)(x_1 \cdot x_2 - 1)}{x_1 \cdot x_2} > 0$, поэтому $y_1 > y_2$

Тогда т.к. $x_1 < x_2$, а $y_1 > y_2$,
 функция убывает на интервале $0 < x < 1$.

$$2) y = \frac{1}{x^2 + 1}; y \text{ возрастает при } x \in (-\infty; 0],$$

y убывает при $x \in [0; +\infty)$.



$$3) y = x^3 - 3x.$$

Пусть $x_1 < x_2$ и $x_1, x_2 \leq -1$, значит $y_1 = x_1^3 - 3x_1$; $y_2 = x_2^3 - 3x_2$

$$\begin{aligned} \text{Тогда } y_1 - y_2 &= x_1^3 - 3x_1 - 3x_2 = (x_1^3 - x_2^3) - 3(x_1 - x_2) = \\ &= (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) - 3(x_1 - x_2) = (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2 - 3) < 0 \end{aligned}$$

при $x_1 \leq -1$, $x_2 \leq -1$, имеем $x_1^2 + x_1x_2 + x_2^2 \geq 3$, поэтому $x_1^2 + x_1x_2 + x_2^2 - 3 \geq 0$,

значит, т.к. $x_1 < x_2$ и $y_1 < y_2$, то y возрастает при $x \leq -1$, и $x \geq 1$ и убывает при $-1 \leq x \leq 1$.

$$4) y = x - 2\sqrt{x}; \text{ пусть } x_1 < x_2 \text{ и } x_1, x_2 \geq 1, \text{ тогда}$$

$$\begin{aligned} y_1 - y_2 &= (x_1 - x_2) - 2(\sqrt{x_1} - \sqrt{x_2}) = (\sqrt{x_1} - \sqrt{x_2})(\sqrt{x_1} + \sqrt{x_2}) - \\ &- 2(\sqrt{x_1} - \sqrt{x_2}) = (\sqrt{x_1} - \sqrt{x_2})(\sqrt{x_1} + \sqrt{x_2} - 2) < 0, \end{aligned}$$

при $x_1 \geq 1$, $x_2 \geq 1$, имеем: $\sqrt{x_1} \geq 1$, $\sqrt{x_2} \geq 1$, значит, $\sqrt{x_1} + \sqrt{x_2} \geq 2$
 поэтому, т.к. $x_1 < x_2$ и $y_1 < y_2$, то y возрастает при $x \geq 1$, убывает при $0 \leq x < 1$.

171.

$$1) y = \begin{cases} x+2, & x \leq -1 \\ x^2, & x > -1 \end{cases};$$

y возрастает при $x \in (-\infty, -1] \cup [0, +\infty)$,

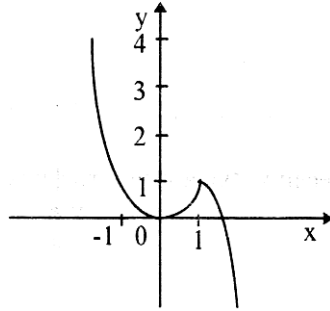
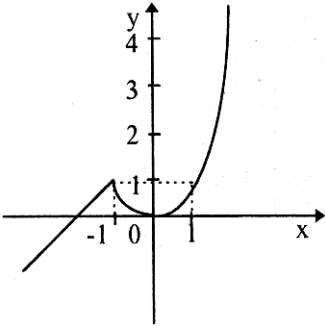
y убывает при $x \in [-1, 0]$.

$$2) y = \begin{cases} x^2, & x \leq 1 \\ 2-x^2, & x > 1 \end{cases};$$

y возрастает при $x \in [0, 1]$,

y убывает

при $x \in (-\infty, 0] \cup [1, +\infty)$.



172.

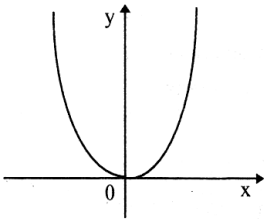
- 1) $y = 2x^4$ – четная, т.к. $y(-x) = 2(-x)^4 = 2x^4 = y(x)$;
- 2) $y = 3x^5$ – нечетная, т.к. $y(-x) = 3(-x)^5 = -3x^5 = -y(x)$;
- 3) $y = x^2 + 3$ – четная, т.к. $y(-x) = (-x)^2 + 3 = x^2 + 3 = y(x)$;
- 4) $y = x^3 - 2$ – не является ни четной, ни нечетной, т.к.
 $y(-x) = (-x)^3 - 2 = -x^3 - 2 \neq -x^3 + 2 = -y(x)$,
 $y(-x) = -x^3 - 2 \neq x^3 - 2 = y(x)$.

173.

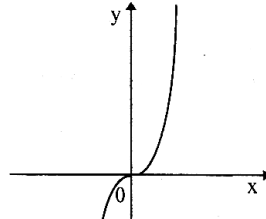
- 1) $y = x^{-4}$ – четная;
- 2) $y = x^{-3}$ – нечетная;
- 3) $y = x^4 + x^2$ – четная;
- 4) $y = x^3 + x^5$ – нечетная;
- 5) $y = x^{-2} - x + 1$ – ни четная ни нечетная;
- 6) $y = \frac{1}{x+1}$ – ни четная ни нечетная.

174.

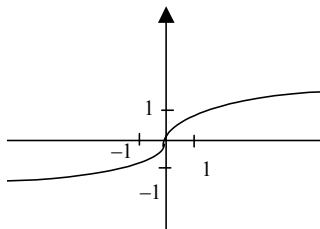
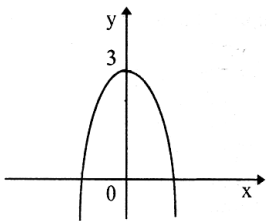
- 1) $y = x^4$;
- 2) $y = x^5$;



3) $y = -x^2 + 3$;



4) $y = \sqrt[5]{x}$.



175.

$$1) y(x) = \frac{x+2}{x-3};$$

$$y(x) \neq y(-x),$$

$$y(-x) = \frac{-x+2}{-x-3} = \frac{-(x-2)}{-(x+3)} = \frac{x-2}{x+3};$$

$$y(x) \neq -y(-x),$$

поэтому $y(x)$ ни четная, ни нечетная.

$$2) y(x) = \frac{x^2+x-1}{x+4};$$

$$y(x) \neq y(-x),$$

$$y(-x) = \frac{x^2-x-1}{-x+4} = \frac{x^2-x-1}{-(x-4)};$$

$$y(x) \neq -y(-x),$$

значит $y(x)$ ни четная, ни нечетная.

176.

$$1) y = x^4 + 2x^2 + 3 - \text{четная};$$

$$2) y = x^3 + 2x + 1 - \text{ни четная, ни нечетная};$$

$$3) y = \frac{3}{x^3} + \sqrt[3]{x},$$

$$y(-x) = \frac{3}{-x^3} + \sqrt[3]{-x} = -\left(\frac{3}{x^3} + \sqrt[3]{x}\right) = -y(x), \text{ т.е. нечетная};$$

$$4) y = x^4 + |x| - \text{четная};$$

$$5) y = |x| + x^3 - \text{ни четная, ни нечетная};$$

$$6) y = \sqrt[3]{x-1} - \text{ни четная, ни нечетная.}$$

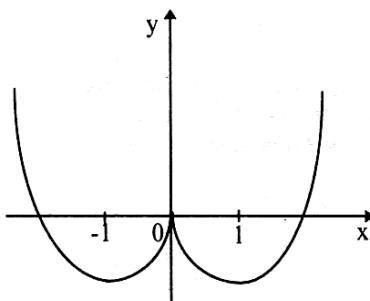
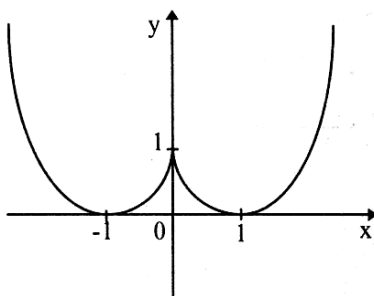
177.

$$1) y = x^2 - 2|x| + 1;$$

$$2) y = x^2 - 2|x|;$$

$$y = \begin{cases} x^2 - 2x + 1, & x \geq 0; \\ x^2 + 2x + 1, & x < 0 \end{cases};$$

$$y = \begin{cases} x^2 - 2x, & x \geq 0 \\ x^2 + 2x, & x < 0 \end{cases}.$$



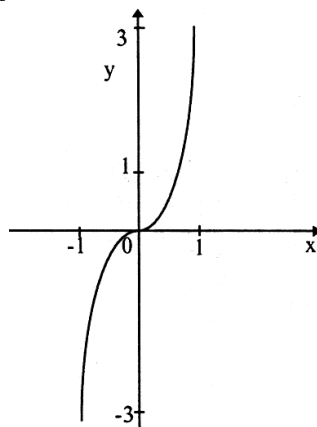
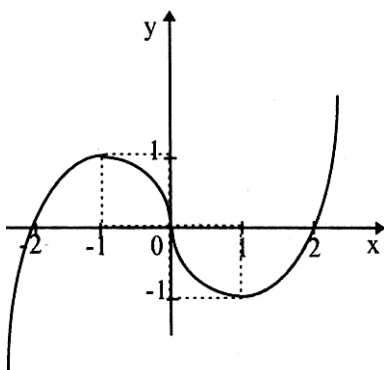
178.

1) $y = x|x| - 2x$;

$$y = \begin{cases} x^2 - 2x & x \geq 0 \\ -x^2 - 2x, & x < 0 \end{cases};$$

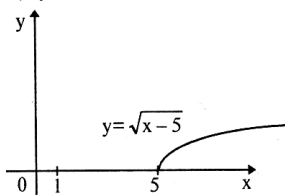
2) $y = x|x| + 2x$;

$$y = \begin{cases} x^2 + 2x, & x \geq 0 \\ -x^2 + 2x, & x < 0 \end{cases}$$



179.

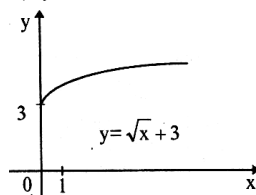
1) $y = \sqrt{x-5}$;



определена при $x - 5 \geq 0, x \geq 5$;

$y = \sqrt{x-5}$ – ни четная,

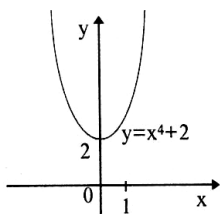
2) $y = \sqrt{x} + 3$;



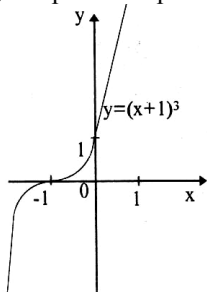
определена при $x \geq 0$;

$y = \sqrt{x} + 3$ – ни четная,

ни нечетная;
 y возрастает, если $x \geq 5$;
 3) $y = x^4 + 2$;
 определена при любом x ;
 $y = x^4 + 2$ – четная;
 y убывает, если $x \in (-\infty; 0)$;
 y возрастает, если $x \in (0; +\infty)$;



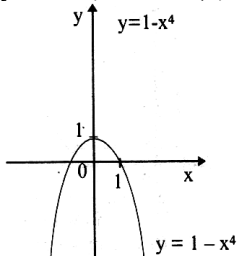
5) $y = (x + 1)^3$;
 определена при $x \in (-\infty; \infty)$;
 $y = (x + 1)^3$ – ни четная,
 ни нечетная;
 y возрастает при всех x ;



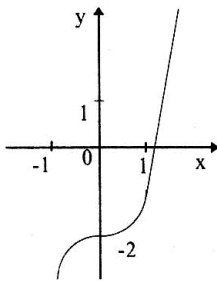
180.

$$1) y = \begin{cases} x^2, & \text{если } x \geq 0 \\ x^3, & \text{если } x < 0 \end{cases};$$

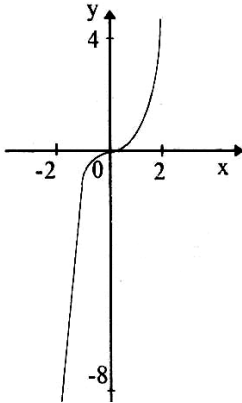
ни нечетная;
 y возрастает, если $x \geq 0$;
 4) $y = 1 - x^4$;
 определена при $x \in (-\infty; \infty)$;
 $y = 1 - x^4$ – четная;
 y возрастает, если $x \in (-\infty; 0)$;
 y убывает, если $x \in (0; +\infty)$;



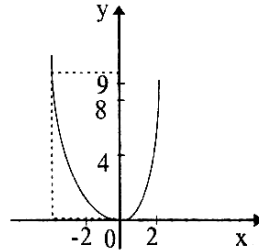
6) $y = x^3 - 2$;
 определена при $x \in (-\infty; \infty)$;
 $y = x^3 - 2$ – ни четная,
 ни нечетная;
 y возрастает при всех x .



$$2) y = \begin{cases} x^3, & \text{если } x > 0 \\ x^2, & \text{если } x \leq 0 \end{cases};$$



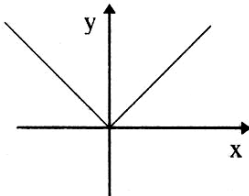
- а) $y > 0$, если $x > 0$;
 б) y возрастает, если $x \in (-\infty; \infty)$;



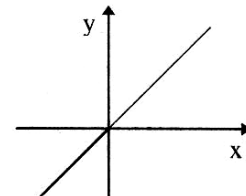
- а) $y > 0$, если $x \neq 0$;
 б) y убывает, если $x \in (-\infty; 0)$;
 y возрастает, если $x \in (0; +\infty)$.

181.

1) $y = x; x > 0$;

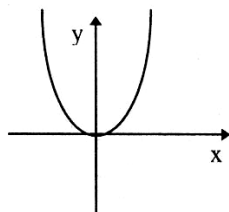


- а) пусть y – четная, тогда $y = |x|$;

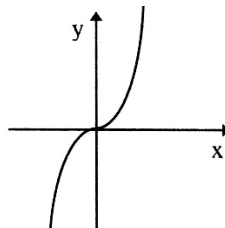


- б) пусть y – нечетная, тогда $y = x$;

2) $y = x^2; x > 0$;

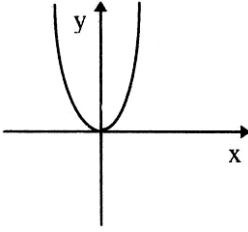


- а) пусть y – четная, тогда $y = x^2$;

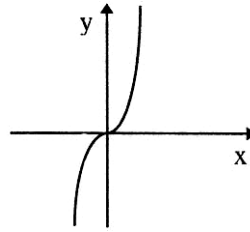


- б) пусть y – нечетная, тогда $y = x|x|$;

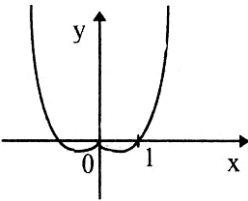
3) $y = x^2 + x; x > 0$;



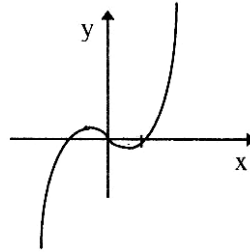
- а) пусть y – четная,
тогда $y = x^2 + |x|$;
4) $y = x^2 - x$; $x > 0$;



- б) пусть y – нечетная,
тогда $y = x|x| + x$;



- а) пусть y – четная,
тогда $y = x^2 - |x|$;



- б) пусть y – нечетная, тогда
 $y = x|x| - x$.

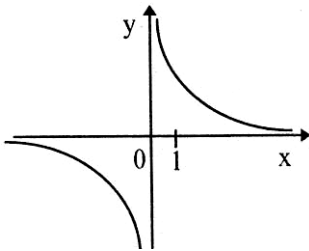
182.

- 1) $y = (x + 1)^6$; ось симметрии: $x = -1$;
2) $y = x^6 + 1$; ось симметрии: $x = 0$.

183.

- 1) $y = x^3 + 1$
центр симметрии: т.М (0,1);
2) $y = (x + 1)^3$
центр симметрии: т.М (-1,0).

184.



$$y = \frac{2}{x};$$

- 1) $y(x) = 4$, если $x = \frac{1}{2}$;
2) $y(x) = -\frac{1}{2}$, если $x = -4$;
3) $y(x) > 1$, если $0 < x < 2$;
4) $y(x) \leq 1$, если $x < 0$ и $x \geq 2$.

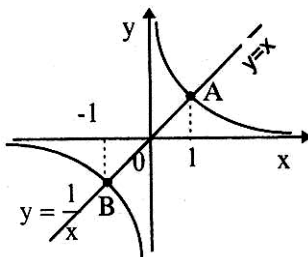
185.

$$y = \frac{1}{x}; y = x;$$

1) в точках $A(1; 1)$ и $B(-1; -1)$;

2) график функции $y = \frac{1}{x}$ лежит

выше, чем график $y = x$, если $x < -1$ и $0 < x < 1$, и ниже, если $-1 < x < 0$ и $x > 1$.



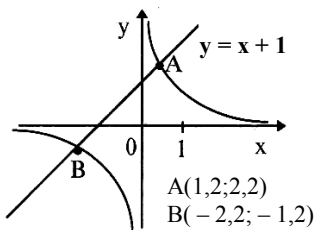
186.

$$1) \begin{cases} y = \frac{12}{x}, \text{ точки } (2;6); (-2;-6); \\ y = 3x \end{cases} \quad 2) \begin{cases} y = -\frac{8}{x}, \text{ точки } (2;-4); (-2;4); \\ y = -2x \end{cases}$$

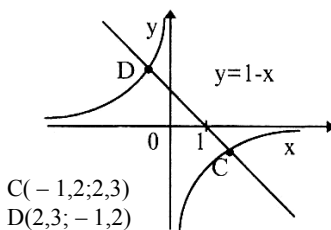
$$3) \begin{cases} y = \frac{2}{x}, \text{ точки } (2;1); (-1;-2); \\ y = x - 1 \end{cases} \quad 4) \begin{cases} y = \frac{6}{x+1}, \text{ точки } (1;3); (-4;-2). \\ y = x + 2 \end{cases}$$

187.

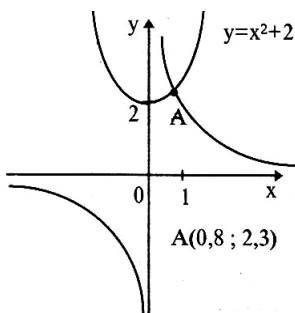
1) $y = \frac{3}{x}; y = x + 1;$



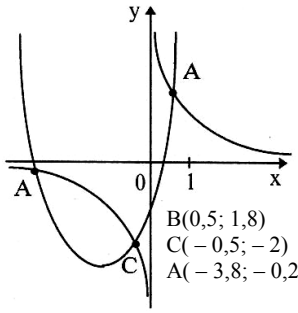
2) $y = -\frac{3}{x}; y = 1 - x;$



3) $y = \frac{2}{x}; y = x^2 + 2;$



4) $y = \frac{1}{x}; y = x^2 + 4x.$



188.

$$V = \frac{12}{\rho}$$

1) $V(4) = \frac{12}{4} = 3$ (л.);

2) $3 = \frac{12}{\rho}$, $\rho = \frac{12}{3}$, $\rho = 4$ (атм);

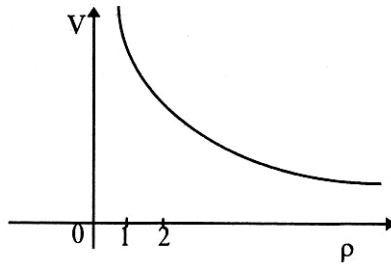
$V(5) = \frac{12}{5} = 2\frac{2}{5}$ (л.);

$5 = \frac{12}{\rho}$, $\rho = \frac{12}{5}$, $\rho = 2\frac{2}{5}$ (атм);

$V(10) = \frac{12}{10} = 1\frac{1}{5}$ (л.);

$15 = \frac{12}{\rho}$, $\rho = \frac{12}{15}$, $\rho = \frac{4}{5}$ (атм).

3)



189.

$$I = \frac{U}{R}; I = \frac{6}{R};$$

1) $R = \frac{6}{10} = 0,6$ (Ом);

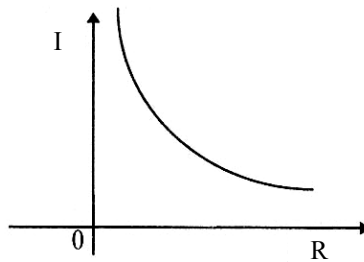
2) $I = \frac{6}{6} = 1$ (А);

$R = \frac{6}{5} = 1\frac{1}{5}$ (Ом);

$I = \frac{6}{12} = \frac{1}{2}$ (А);

$R = \frac{6}{1,2} = 5$ (Ом);

$I = \frac{6}{20} = \frac{3}{10}$ (А).



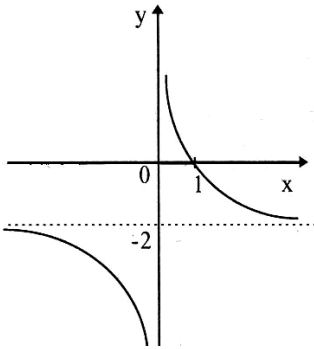
190.

$$a_y = \frac{v^2}{r}; \quad a_y = \frac{60^2}{0,15} = 24000 \text{ км/ч}^2,$$

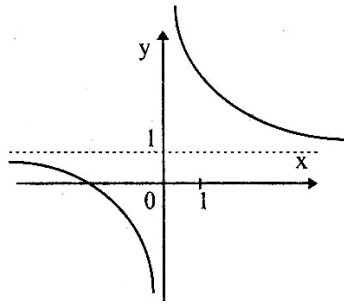
a_y уменьшится, если увеличится радиус.

191.

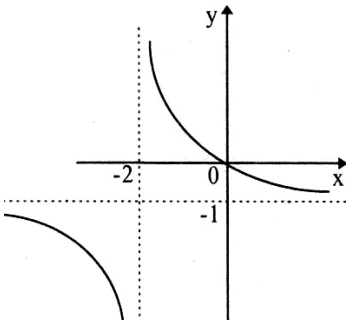
1) $y = \frac{3}{x} - 2;$



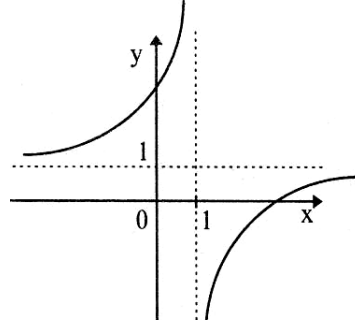
2) $y = \frac{2}{x} + 1;$



3) $y = \frac{2}{x+2} - 1;$



4) $y = \frac{2}{1-x} + 1.$



192.

1) $x^7 > 1$, тогда
 $x > 1$.

Ответ: $x \in (1; \infty)$.

3) $y^3 \geq 64;$
 $y^3 \geq 4^3$, поэтому
 $y \geq 4$.

Ответ: $y \in [4; +\infty)$.

2) $x^3 \leq 27$, значит,
 $x^3 \leq 3^3$, $x \leq 3$.

Ответ: $x \in (-\infty; 3]$.

4) $y^3 < 125;$
 $y^3 < 5^3$, значит,
 $y < 5$.

Ответ: $y \in (-\infty; 5)$.

$$5) x^4 \leq 16;$$

$$(x^2 - 4)(x^2 + 4) \leq 0, \text{ значит,}$$

$$(x - 2)(x + 2)(x^2 + 4) \leq 0.$$

Ответ: $x \in [-2; 2]$.

193.

1) $S = a^2$, и $a^2 > 361$
 a – сторона квадрата,
 значит, $a > 0$;

2) $V = a^3$, т.е.
 a – ребро куба,
 тогда $a > 0$

194.

1) $\sqrt{x-3} = 2$;
 $\sqrt{7-3} = 2$;
 $\sqrt{4} = 2$,
 значит, 7 – корень;

195.

1) $\sqrt{x} = 3$; 2) $\sqrt{x} = 7$;
 $x = 3^2 = 9$; $x^2 = 7^2 = 49$;

196.

1) $\sqrt{x+1} = 2$ по О.Д.З.
 $x + 1 = 4$; $x \geq -1$,
 $x = 3$ входит в О.Д.З.;
 3) $\sqrt{1-2x} = 4$, по О.Д.З.
 $1 - 2x = 16$; $x \leq \frac{1}{2}$; $-2x = 15$;
 $x = -7,5$ входит в О.Д.З.;

$$6) x^4 > 625;$$

$$(x^2 - 25)(x^2 + 25) > 0, \text{ тогда}$$

$$(x - 5)(x + 5)(x^2 + 25) > 0.$$

Ответ: $x \in (-\infty; -5) \cup (5; +\infty)$.

$a^2 - 361 > 0$,
 $(a - 19)(a + 19) > 0, a > 0$.



Ответ: $a > 19(\text{см})$.
 $a^3 > 343$;
 $a^3 > 7^3$;
 $a > 7$, значит $a > 7(\text{см})$.
 Ответ: $a > 7(\text{см})$.

2) $\sqrt{x^2-13} - \sqrt{2x-5} = 3$;
 $\sqrt{49-13} - \sqrt{14-5} = 6-3=3$,
 поэтому
 7 – корень.

3) $\sqrt{2x-1} = 0$; 4) $\sqrt{3x+2} = 0$;
 $2x - 1 = 0$; $3x + 2 = 0$;
 $x = \frac{1}{2}$; $x = -\frac{2}{3}$.

2) $\sqrt{x-1} = 3$ по О.Д.З.
 $x - 1 = 9$; $x \geq 1$,
 $x = 10$ входит в О.Д.З.;
 4) $\sqrt{2x-1} = 3$, по О.Д.З.;
 $2x - 1 = 9$; $x \geq \frac{1}{2}$; $2x = 10$;
 $x = 5$ входит в О.Д.З.

197.

$$1) \sqrt{x+1} = \sqrt{2x-3} \text{ по О.Д.З. } \begin{cases} x \geq -1 \\ x \geq 1,5 \end{cases} x \geq 1,5;$$

$$x + 1 = 2x - 3;$$

$$x = 4 \text{ входит в О.Д.З.}$$

$$\text{Ответ: } x = 4.$$

$$2) \sqrt{x-2} = \sqrt{3x-6} \text{ по О.Д.З. } x \geq 2$$

$$\sqrt{x-2} = \sqrt{3(x-2)}$$

$$x = 2 \text{ входит в О.Д.З.}$$

$$\text{Ответ: } x = 2.$$

$$3) \sqrt{x^2 + 24} = \sqrt{11x} \text{ по О.Д.З. } x \geq 0;$$

$$x^2 + 24 = 11x$$

$$x^2 - 11x + 24 = 0, x_1 = 3 \text{ и } x_2 = 8 \text{ входят в О.Д.З.}$$

$$\text{Ответ: } x_1 = 3; x_2 = 8.$$

$$4) \sqrt{x^2 + 4x} = \sqrt{14-x}$$

$$\text{по О.Д.З. } \begin{cases} x \leq 14 \\ x^2 + 4x \geq 0 \end{cases} \left| x \in (-\infty; -4] \cup [0; 14] \right.;$$

$$x^2 + 4x + x - 14 = 0;$$

$$x^2 + 5x - 14 = 0,$$

$$x_1 = 2 \text{ и } x_2 = -7 \text{ входят в О.Д.З.}$$

$$\text{Ответ: } x_1 = 2; x_2 = -7.$$

198.

$$1) x + 2 = x^2 \text{ по О.Д.З. } x \geq 0;$$

$$x^2 - x - 2 = 0;$$

$$x_1 = 2; x_2 = -1;$$

$$x_2 = -1 - \text{не входит в О.Д.З.}$$

$$\text{Ответ: } x = 2.$$

$$2) 3x + 4 = x^2 \text{ по О.Д.З. } x \geq 0,$$

$$\begin{cases} x \geq -1\frac{1}{3} \Rightarrow x \geq 0; \\ x \geq 0 \end{cases}$$

$$x^2 - 3x - 4 = 0;$$

$$x_1 = 4; x_2 = -1;$$

$$x_2 = -1 - \text{не входит в О.Д.З., т.к. } -1 < 0.$$

$$\text{Ответ: } x = 4.$$

$$3) \sqrt{20-x^2} = 2x; \text{ О.Д.З. } \begin{cases} 20-x^2 \geq 0; \\ x \geq 0 \end{cases}; \quad x \in [0; 2\sqrt{5}]$$

$$20-x^2 = 4x^2;$$

$$5x^2 = 20;$$

$$x_1 = 2; x_2 = -2, x_2 = -2 - \text{ не входит в О.Д.З., т.к. } -2 < 0.$$

$$\text{Ответ: } x = 2.$$

$$4) \sqrt{0,4-x^2} = 3x; \text{ О.Д.З. } \begin{cases} 0,4-x^2 \geq 0; \\ x \geq 0 \end{cases}; \quad x \in [0; 2\sqrt{0,1}]$$

$$0,4-x^2 = 9x^2$$

$$10x^2 = 0,4; x^2 = 0,04;$$

$$x = 0,2; x = -0,2, x_2 = -0,2 - \text{ не входит в О.Д.З., т.к. } -0,2 < 0.$$

$$\text{Ответ: } x = 0,2.$$

199.

$$1) \sqrt{x^2-x-8} = x-2; \text{ О.Д.З. } \begin{cases} x^2-x-8 \geq 0; \\ x-2 \geq 0 \end{cases}; \quad x \in \left[\frac{1+\sqrt{33}}{2}, +\infty \right)$$

$$x^2-x-8 = x^2-4x+4$$

$$3x = 12, x = 4 \text{ входит в О.Д.З.}$$

$$\text{Ответ: } x = 4.$$

$$2) \sqrt{x^2+x-6} = x-1; \text{ О.Д.З. } \begin{cases} x^2+x-6 \geq 0; \\ x-1 \geq 0 \end{cases}; \quad x \in [2; +\infty);$$

$$x^2+x-6 = x^2-2x+1;$$

$$3x = 7, x = 2\frac{1}{3}, \text{ входит в О.Д.З.}$$

$$\text{Ответ: } x = 2\frac{1}{3}.$$

200.

$$1) (x-1)^3 > 1,$$

$$\text{тогда } x-1 > 1$$

$$\text{и } x > 2.$$

$$\text{Ответ: } x \in (2; +\infty).$$

$$3) (2x-3)^7 \geq 1,$$

$$\text{поэтому } 2x-3 \geq 1$$

$$\text{и } x \geq 2.$$

$$\text{Ответ: } x \in [2; +\infty).$$

$$2) (x+5)^3 > 8,$$

$$\text{значит, } x+5 > 2$$

$$\text{и } x > -3.$$

$$\text{Ответ: } x \in (-3; +\infty).$$

$$4) (3x-5)^7 < 1,$$

$$\text{отсюда } 3x-5 < 1$$

$$\text{и } x < 2.$$

$$\text{Ответ: } x \in (-\infty; 2).$$

5) $(3-x)^4 > 256$; $((3-x)^2 - 16)((3-x)^2 + 16) > 0$
 $(3-x-4)(3-x+4) > 0$, т.к. $(3-x)^2 + 16 > 0$ при любом x ,
 тогда $(-x-1)(7-x) > 0$.



Ответ: $x \in (-\infty; -1) \cup (7; +\infty)$.

6) $(4-x)^4 > 81$; $((4-x)^2 - 9)((4-x)^2 + 9) > 0$,
 т.к. $(4-x)^2 + 9 > 0$, то
 $(4-x-3)(4-x+3) > 0$,
 тогда $(1-x)(7-x) > 0$.



Ответ: $x \in (-\infty; 1) \cup (7; +\infty)$.

201.

1) $\sqrt{x} = -8$ — не имеет смысла, т.к. $\sqrt{x} \geq 0$;

2) $\sqrt{x} + \sqrt{x-4} = -3$ — не имеет смысла, т.к. слева стоит сумма неотрицательных слагаемых, а справа отрицательное число;

3) $\sqrt{-2-x^2} = 12$ — не имеет смысла, т.к. $-2-x^2 < 0$

для любого x ;

4) $\sqrt{7x-x^2-63} = 5$ не имеет смысла, т.к.
 $7x-x^2-63 < 0$

для любых x .

202.

1) $\sqrt{x^2+4x+9} = 2x-5$; О.Д.З. $\begin{cases} x^2-4x+9 \geq 0 \\ 2x-5 \geq 0 \end{cases}$; $x \in \left[\frac{5}{2}; +\infty\right)$

возводим в квадрат $x^2 - 4x + 9 = 4x^2 - 20x + 25$

$3x^2 - 16x + 16 = 0$. Решим:

$$\frac{D}{4} = 8^2 - 3 \cdot 16 = 64 - 48 = 16;$$

$$x_{1,2} = \frac{8 \pm 4}{3}, x_1 = 4 \text{ входит в О.Д.З.};$$

$$x_2 = 1\frac{1}{3} \text{ не входит в О.Д.З.}$$

Ответ: $x = 4$.

$$2) \sqrt{x^2+3x+6} = 3x+8; \quad \text{О.Д.З.} \begin{cases} x^2+3x+6 \geq 0; \\ 3x+8 \geq 0 \end{cases}; \quad x \in \left[-2\frac{2}{3}; +\infty\right)$$

возведем в квадрат $x^2+3x+6 = 9x^2+48x+64$;

$$8x^2+45x+58 = 0. \text{ Решим:}$$

$$D = 2025 - 1856 = 169 > 0,$$

$$x_{1,2} = \frac{-45 \pm 13}{16};$$

$$x_1 = \frac{-58}{16} = -\frac{29}{8} = -3\frac{5}{8} \text{ не входит в О.Д.З.};$$

$$x_2 = \frac{-32}{16} = -2 \text{ входит в О.Д.З.}$$

Ответ: $x = -2$.

$$3) 2x = 1 + \sqrt{x^2+5}; \quad \text{О.Д.З.} \quad 2x-1 \geq 0, \quad x \in \left[\frac{1}{2}; +\infty\right);$$

$\sqrt{x^2+5} = 2x-1$. Возводим в квадрат $x^2+5 = 4x^2-4x+1$
 $3x^2-4x-4 = 0$. Решим:

$$\frac{D}{4} = 4+12 = 16;$$

$$x_1 = \frac{2 \pm 4}{3}, \quad x_1 = 2 \text{ - входит в О.Д.З.}; \quad x_2 = -\frac{2}{3} \text{ - не входит в О.Д.З.}$$

Ответ: $x = 2$.

$$4) x + \sqrt{13-4x} = 4; \quad \text{О.Д.З.} \begin{cases} 13-4x \geq 0; \\ 4-x \geq 0 \end{cases}; \quad x \in \left(-\infty; 3\frac{1}{4}\right];$$

$\sqrt{13-4x} = 4-x$. Возведем в квадрат
 $13-4x = 16-8x+x^2; x^2+4x=3=0$. Решим:

$$x_1 = 3, x_2 = 1 \text{ входят в О.Д.З.}$$

Ответ: $x_1 = 3; x_2 = 1$.

203.

$$1) \sqrt{x+12} = 2 + \sqrt{x}; \quad \text{О.Д.З.} \begin{cases} x \geq 0 \\ x+12 \geq 0 \end{cases}; \quad x \in [0; +\infty);$$

возводим в квадрат $x+12 = 4+4\sqrt{x}+x$;

$$4\sqrt{x} = 8; \quad \sqrt{x} = 2; \quad x = 4 \text{ входит в О.Д.З.}$$

Ответ: $x = 4$.

$$2) \sqrt{4+x} + \sqrt{x} = 4; \text{ О.Д.З. } \begin{cases} x \geq 0 \\ 4+x \geq 0 \end{cases} x \in [0; +\infty);$$

$$\sqrt{4+x} = 4 - \sqrt{x}. \text{ Возводим в квадрат}$$

$$4+x = 16 - 8\sqrt{x} + x;$$

$$-8\sqrt{x} = -12;$$

$$\sqrt{x} = 1,5, x = 2,25 \text{ входит в О.Д.З.}$$

Ответ: $x = 2,25$.

204.

$$1) \sqrt{2x+1} + \sqrt{3x+4} = 3; \text{ О.Д.З. } \begin{cases} 2x+1 \geq 0 \\ 3x+4 \geq 0 \end{cases}; x \in \left[-\frac{1}{2}; +\infty\right)$$

$$\sqrt{3x+4} = 3 - \sqrt{2x+1}, \text{ возводим в квадрат}$$

$$3x+4 = 9 - 6\sqrt{2x+1} + 2x+1; x-6 = -6\sqrt{2x+1};$$

$$6\sqrt{2x+1} = 6-x; \text{ О.Д.З. } 6-x \geq 0,$$

возводим в квадрат $36(2x+1) = 36 - 12x + x^2$;

$$x \leq 6, \text{ т.е. } x \in \left[-\frac{1}{2}; 6\right] - \text{общая О.Д.З.};$$

$$72x + 36 = 36 - 12x + x^2;$$

$$x^2 - 84x = 0. \text{ Решим: } x(x-84) = 0, x_1 = 0 \text{ входит в О.Д.З.};$$

$$x_2 = 84 \text{ не входит в О.Д.З.}$$

Ответ: $x = 0$.

$$2) \sqrt{4x-3} + \sqrt{5x+4} = 4; \text{ О.Д.З. } \begin{cases} 4x-3 \geq 0 \\ 5x+4 \geq 0 \end{cases}; x \in \left[\frac{3}{4}; +\infty\right)$$

$$\sqrt{5x+4} = 4 - \sqrt{4x-3}, \text{ возводим в квадрат}$$

$$5x+4 = 16 - 8\sqrt{4x-3} + 4x-3$$

$$x-9 = -8\sqrt{4x-3} \text{ запишем еще один О.Д.З. } 9-x \geq 0,$$

возводим в квадрат $x^2 - 18x + 81 = 64(4x+3)$;

$$x \leq 9, \text{ т.е. } x \in \left[\frac{3}{4}; 9\right] - \text{общая О.Д.З.};$$

$$x^2 - 18x + 81 = 256x - 192;$$

$$x^2 - 274x + 273 = 0. \text{ Решим:}$$

$$x_1 = 273, x_2 = 1; x_1 = 273 - \text{не входит в О.Д.З.},$$

$$x_1 = 1 - \text{входит в О.Д.З.}$$

Ответ: $x = 1$.

$$3) \sqrt{x-7} - \sqrt{x+17} = -4; \quad \text{О.Д.З.} \begin{cases} x-7 \geq 0 \\ x+17 \geq 0 \end{cases}; \quad x \in [7; +\infty);$$

$$\sqrt{x+17} = \sqrt{x-7} + 4, \text{ возводим в квадрат}$$

$$x+17 = 16 + 8\sqrt{x-7} + x-7$$

$$8 = 8\sqrt{x-7}$$

$$1 = \sqrt{x-7}, x-7 = 1,$$

$$x = 8 \text{ входит в О.Д.З.}$$

$$\text{Ответ: } x = 8.$$

$$4) \sqrt{x+4} - \sqrt{x-1} = 1; \quad \text{О.Д.З.} \begin{cases} x+4 \geq 0 \\ x-1 \geq 0 \end{cases}; \quad x \in [1; +\infty);$$

$$\sqrt{x+4} = 1 + \sqrt{x-1}, \text{ возводим в квадрат}$$

$$x+4 = 1 + 2\sqrt{x-1} + x-1;$$

$$4 = 2\sqrt{x-1};$$

$$2 = \sqrt{x-1}, x-1 = 4,$$

$$x = 5 \text{ входит в О.Д.З.}$$

$$\text{Ответ: } x = 5.$$

205.

$$1) \sqrt{4+\sqrt{x}} = \sqrt{19-2\sqrt{x}}; \quad \text{О.Д.З.} \begin{cases} x \geq 0 \\ 19-2\sqrt{x} \geq 0 \end{cases}; \quad x \in \left[0; 90\frac{1}{4}\right];$$

$$\text{возводим в квадрат } 4 + \sqrt{x} = 19 - 2\sqrt{x};$$

$$3\sqrt{x} = 15,$$

$$\text{тогда } \sqrt{x} = 5;$$

$$x = 25 - \text{входит в О.Д.З.}$$

$$\text{Ответ: } x = 25.$$

$$2) \sqrt{7+\sqrt{x}} = \sqrt{11-\sqrt{x}}; \quad \text{О.Д.З.} \begin{cases} x \geq 0 \\ 11-\sqrt{x} \geq 0 \end{cases}; \quad x \in [0; 121];$$

возводим в квадрат

$$7 + \sqrt{x} = 11 - \sqrt{x}$$

$$2\sqrt{x} = 4;$$

$$\sqrt{x} = 2;$$

$$x = 4 - \text{входит в О.Д.З.}$$

$$\text{Ответ: } x = 4.$$

206.

1) $\sqrt{x-2} > 3$; О.Д.З.

и возведем в квадрат

$$\begin{cases} x-2 \geq 0 \\ x-2 > 9 \end{cases}; \begin{cases} x \geq 2 \\ x > 11 \end{cases}; x > 11$$

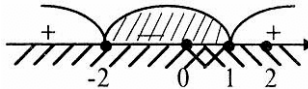
Ответ: $x \in (11; +\infty)$.

2) $\sqrt{x-2} \leq 1$; $\begin{cases} x-2 \geq 0 \\ x-2 \leq 1 \end{cases}; \begin{cases} x \geq 2 \\ x \leq 3 \end{cases};$

$2 \leq x \leq 3$.

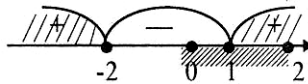
Ответ: $x \in [2; 3]$.

3) $\sqrt{2-x} \geq x$; $\begin{cases} 2-x \geq 0 \\ 2-x \geq x^2 \end{cases}; \begin{cases} x \leq 2 \\ x^2+x-2 \leq 0 \end{cases}; \begin{cases} x \leq 2 \\ (x+2)(x-1) \leq 0 \end{cases}$



Ответ: $x \in (-\infty; 1]$.

4) $\sqrt{2-x} < x$; $\begin{cases} 2-x \geq 0 \\ x \geq 0 \\ 2-x < x^2 \end{cases}; \begin{cases} x \leq 2 \\ x \geq 0 \\ x^2+x-2 > 0 \end{cases}; \begin{cases} x \leq 2 \\ x \geq 0 \\ x < -2 \text{ или } x > 1 \end{cases}$



Ответ: $x \in (1; 2]$.

5) $\sqrt{5x+11} > x+3$; $\begin{cases} 5x \geq 0 \\ 5x+11 > x^2+6x+9 \end{cases}; \begin{cases} x \geq 2,2 \\ x^2+x-2 < 0 \end{cases}$

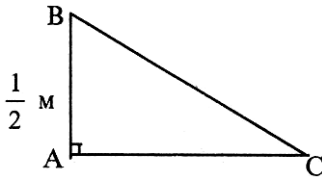


Ответ: $x \in (-2; 1)$

6) $\sqrt{x+3} \leq x+1$; $\begin{cases} x+3 \geq 0 \\ x+1 \geq 0 \\ x+3 \leq x^2+2x+1 \end{cases}; \begin{cases} x \geq -3 \\ x \geq -1 \\ x^2+x-2 \geq 0 \end{cases}$



Ответ: $x \in [1; +\infty)$.



207.

$$BC - AC \leq 0,02.$$

Если $AC = x$,

$$\text{то } BC = \sqrt{x^2 + \frac{1}{4}}.$$

$$\text{Получим } \sqrt{x^2 + \frac{1}{4}} - x \leq 0,02;$$

$$\sqrt{x^2 + \frac{1}{4}} \leq 0,02 + x; \text{ О.Д.З.};$$

$$\begin{cases} 0,02 + x \geq 0 \\ x^2 + \frac{1}{4} \leq 0,0004 + 0,04x + x^2. \end{cases}$$

$$\text{Возведем в квадрат}$$

$$\begin{cases} x \geq -0,02 \\ 0,04x \geq 0,2496 \end{cases} \cdot \begin{cases} x \geq -0,02 \\ x \geq 6,24 \end{cases}.$$

Ответ: на расстоянии $\geq 6,24$ (м).

208.

$$1) y = \frac{1}{2x+1}, \text{ значит, } 2x+1 \neq 0,$$

$$x \neq -\frac{1}{2}, \text{ тогда } x \in \left(-\infty; -\frac{1}{2}\right) \cup \left(-\frac{1}{2}; \infty\right);$$

$$2) y = (3-2x)^{-2}, \text{ тогда } 3-2x \neq 0,$$

$$x \neq 1,5, \text{ значит } x \in (-\infty; 1,5) \cup (1,5; \infty);$$

$$3) y = \sqrt{-5-3x}, \text{ значит } -5-3x \geq 0;$$

$$-3x \geq 5;$$

$$x \leq -1\frac{2}{3}, \text{ тогда } x \in \left(-\infty; -1\frac{2}{3}\right];$$

$$4) y = \sqrt[3]{7-3x},$$

имеет смысл для любого x , т.е. $x \in (-\infty; \infty)$.

209.

$$1) \sqrt[4]{2,7} < \sqrt[4]{2,9}, \text{ т.к. } 2,7 < 2,9 \text{ и } \sqrt[4]{x} \text{ - возрастает;}$$

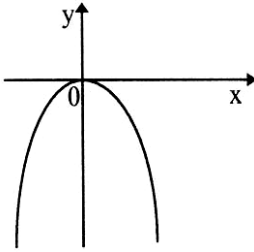
$$2) \sqrt[4]{\frac{1}{7}} > \sqrt[4]{\frac{1}{8}}, \text{ т.к. } \frac{1}{7} > \frac{1}{8} \text{ и } \sqrt[4]{x} \text{ - возрастает;}$$

3) $(-2)^5 > (-3)^5$ т.к. $y = x^5$ – возрастает и $-2 > -3$;

4) $\left(2\frac{2}{3}\right)^5 < \left(2\frac{3}{4}\right)^5$ т.к. $y = x^5$ – возрастает и $2\frac{2}{3} < 2\frac{3}{4}$.

210.

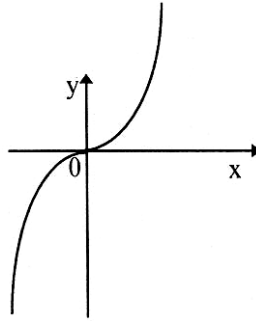
1) $y = -2x^4$;



y – четная;

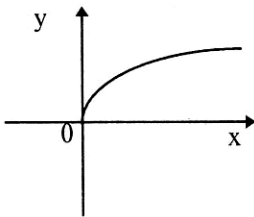
y возрастает, если $x \in (-\infty; 0)$, y возрастает для любого x ;
 y убывает, если $x \in (0; +\infty)$;

2) $y = \frac{1}{2}x^5$;



y – нечетная;

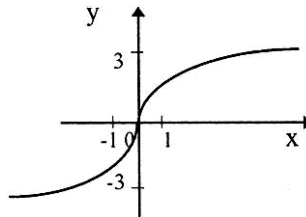
3) $y = 2\sqrt[4]{x}$;



определена при $x \geq 0$;

y – ни чётная, ни нечётная; y – возрастает при всех значениях x .
 y – возрастает при всех x ;

4) $y = 3\sqrt[3]{x}$;



y – нечётная;

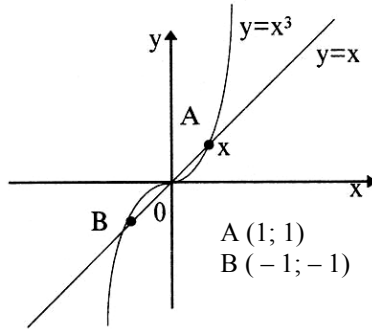
211.

$y = \frac{k}{x}$, если $k = -4$ расположены во II и IV квадрантах,

т.к. $-4 < 0$;

$y = \frac{k}{x}$, если $k = 3$ расположены в I и III квадрантах, т.к. $3 > 0$.

212.



213.

$$1) \begin{cases} y = x^2 \\ y = x^3 \end{cases}; \quad x^2 = x^3.$$

Тогда $x^2 - x^3 = 0$;

$$x^2(x-1) = 0;$$

$x_1 = 0; x_2 = 1$. Точки $A(0; 0)$; $B(1; 1)$.

$$2) \begin{cases} y = \frac{1}{x} \\ y = 2x \end{cases}; \quad \frac{1}{x} = 2x.$$

Тогда $\frac{1-2x}{x} = 0$;

$$1 - 2x^2 = 0;$$

$$x^2 = \frac{1}{2};$$

$x_1 = \frac{\sqrt{2}}{2}; x_2 = -\frac{\sqrt{2}}{2}$, точки $M\left(\frac{\sqrt{2}}{2}; \sqrt{2}\right)$; $N\left(-\frac{\sqrt{2}}{2}; -\sqrt{2}\right)$;

$$3) \begin{cases} y = \sqrt{x} \\ y = |x| \end{cases}; \quad \sqrt{x} = |x|.$$

Значит, $x_1 = 0; x_2 = 1$, точки $M(0; 0)$, $N(1; 1)$;

$$4) \begin{cases} y = \sqrt[3]{x} \\ y = \frac{1}{x} \end{cases}; \quad \sqrt[3]{x} = \frac{1}{x}; \quad x^{\frac{4}{3}} = 1.$$

Получим $x_1 = 1; x_2 = -1$, точки $M(1; 1)$, $N(-1; -1)$.

214.

1) $x^4 \leq 81$;

$(x^2 - 9)(x^2 + 9) \leq 0$, т.к. $x^2 + 9 > 0$, то

$(x - 3)(x + 3) \leq 0$.



Ответ: $x \in [-3; 3]$.

2) $x^5 > 32$;

$x^5 > 2^5$, значит

$x > 2$.

Ответ: $x \in (2; +\infty)$.

3) $x^6 > 64$;

$x^2 > 4$;

$x^2 - 4 > 0$, тогда

$(x - 2)(x + 2) > 0$;

$x > 2$ или $x < -2$.



Ответ: $x \in (-\infty; -2) \cup (2; +\infty)$.

4) $x^5 \leq -32$;

$x^5 \leq (-2)^5$, получим

$x \leq -2$.

Ответ: $x \in (-\infty; -2]$.

215.

1) $\sqrt{3-x} = 2$ по О.Д.З.;

$3-x = 4$; $x \leq 3$;

$x = -1$ входит в О.Д.З.

Ответ: $x = -1$.

2) $\sqrt{3x+1} = 7$ по О.Д.З.;

$3x+1 = 49$ $3x+1 \geq 0$, $x \geq -\frac{1}{3}$;

$3x = 48$;

$x = 16$ входит в О.Д.З.

Ответ: $x = 16$.

3) $\sqrt{3-11x} = 2x$ по О.Д.З. $\begin{cases} x \geq 0 \\ 3-11x \geq 0 \end{cases}$;

возводим в квадрат $3-11x = 4x^2$; $0 \leq x \leq \frac{3}{11}$;

$4x^2 + 11x - 3 = 0$. Решим:

$x_{1,2} = \frac{-11 \pm 13}{8}$ $x_1 = \frac{1}{4}$; входит в О.Д.З. $x_2 = -3$ не входит в

О.Д.З.

Ответ: $x = \frac{1}{4}$.

$$4) \sqrt{5x-1+3x^2} = 3x \text{ по О.Д.З. } \begin{cases} x \geq 0 \\ 3x^2 + 5x - 1 \geq 0 \end{cases};$$

возводим в квадрат:

$$3x^2 + 5x - 1 = 9x^2; x \in (0, 2; \infty);$$

$$6x^2 - 5x + 1 = 0. \text{ Решим:}$$

$$D = 25 - 24 = 1 > 0;$$

$$x_{1,2} = \frac{5 \pm 1}{12}; \quad x_1 = \frac{1}{2} \text{ и } x_2 = \frac{1}{3} \text{ входят в О.Д.З.}$$

$$\text{Ответ: } x_1 = \frac{1}{2}; x_2 = \frac{1}{3}.$$

$$5) \sqrt{2x-1} = x-2 \text{ по О.Д.З. } \begin{cases} x-2 \geq 0, x \geq 2 \\ 2x-1 \geq 0 \end{cases}.$$

Возведем в квадрат:

$$2x-1 = x^2 - 4x + 4; x \geq 2;$$

$$x^2 - 6x + 5 = 0.$$

Решим: $x_1 = 5; x_2 = 1$ не входит в О.Д.З.

Ответ: $x = 5$.

$$6) \sqrt{2-2x} = x+3 \text{ по О.Д.З. } \begin{cases} x+3 \geq 0 \\ 2-2x \geq 0 \end{cases}; \begin{cases} x \geq -3 \\ x \leq 1 \end{cases}.$$

Возводим в квадрат:

$$2-2x = x^2 + 6x + 9;$$

$$x^2 + 8x + 7 = 0.$$

Решим:

$x_1 = -7$ не входит в О.Д.З.; $x_2 = -1$ — входит в О.Д.З.

Ответ: -1 .

216.

$$1) y = \sqrt[3]{x^2 + 2x - 15}, \text{ при всех } x \text{ имеет смысл } x \in (-\infty; \infty);$$

$$2) y = \sqrt[4]{13x - 22 - x^2};$$

$$-x^2 + 13x - 22 \geq 0;$$

$$x^2 - 13x + 22 \leq 0.$$

Решим уравнение $x^2 - 13x + 22 = 0$.

Корни $x_1 = 11; x_2 = 2$, тогда $2 \leq x \leq 11$.

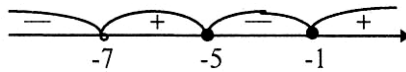


Ответ: $x \in [2; 11]$.

$$3) y = \sqrt{\frac{x^2 + 6x + 5}{x + 7}}$$

Значит, $\frac{x^2 + 6x + 5}{x + 7} \geq 0$. Решим $x^2 + 6x + 5 = 0$;

$x_1 = -1$; $x_2 = -5$; значит, $\frac{(x+1)(x+5)}{x+7} \geq 0$.

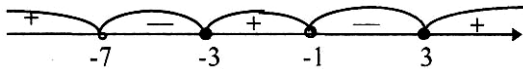


Ответ: $x \in (-7; -5] \cup [-1; +\infty)$.

$$4) y = \sqrt{\frac{x^2 - 9}{x^2 + 8x + 7}}$$

$\frac{x^2 - 9}{x^2 + 8x + 7} \geq 0$. Решим $(x^2 - 9)(x^2 + 8x + 7) = 0$;

$x_1 = 3$; $x_2 = -3$; $x_3 = -7$; $x_4 = -1$ исключая x_3 и x_4 .



Ответ: $x \in (-\infty; -7) \cup [-3; -1) \cup [3; +\infty)$.

217.

$$1) y = \frac{1}{(x-3)^2},$$

y убывает, если $x > 3$;

$$2) y = \frac{1}{(x-2)^3}, x < 2.$$

Если $x_1 = 0$, $x_2 = 1$, $x_1 < x_2$,

то $y(0) = -\frac{1}{8}$; $y_1 > y_2$, тогда

$$y(1) = -1$$

т.к. $x_1 < x_2$, $y_1 > y_2$, то

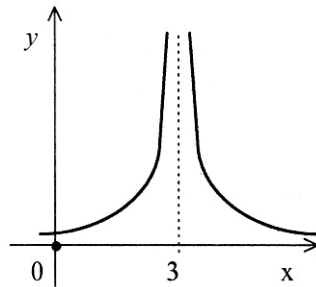
y – убывает, если $x < 2$;

3) $y = \sqrt[3]{x+1}$, $x \geq 0$. Пусть $x_1 = 7$, $x_2 = 26$;

$$y_1 = \sqrt[3]{8} = 2$$

; $y_2 = \sqrt[3]{27} = 3$; $y_1 < y_2$, и т.к. $x_1 < x_2$, то получим, что

y – возрастает, если $x \geq 0$;



$$4) y = \frac{1}{\sqrt[3]{x+1}}, x < -1/$$

Пусть $x_1 = -8, x_2 = -27, x_1 > x_2$;

$$y_1 = \frac{1}{\sqrt[3]{-8}} = -\frac{1}{2};$$

$$y_2 = \frac{1}{\sqrt[3]{-27}} = -\frac{1}{3};$$

получим, что

$y_1 < y_2, x_1 > x_2$, значит y – убывает, если $x < -1$.

218.

$$1) y = x^6 - 3x^4 + x^2 - 2;$$

четная;

$$2) y = x^5 - x^3 + x;$$

нечетная;

$$3) y = \frac{1}{(x-2)^2} + 1;$$

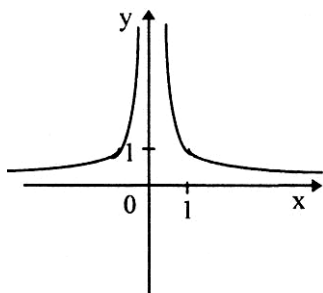
ни четная ни нечетная;

$$4) y = x^7 + x^5 + 1;$$

ни четная ни нечетная/

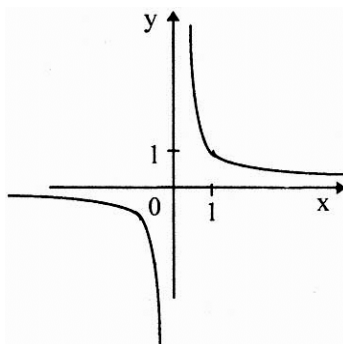
219.

$$1) y = \frac{1}{x^2};$$



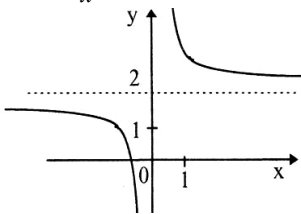
1. y – чётная;
2. y возрастает, если $x \in (-\infty; 0)$;
3. y убывает, если $x \in (0; +\infty)$;

$$2) y = \frac{1}{x^3};$$



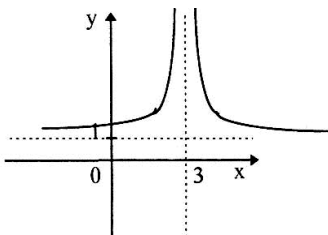
1. y – нечетная;
2. y убывает, если $x \in (-\infty; 0) \cup (0; +\infty)$;

$$3) y = \frac{1}{x^3} + 2;$$



1. y – ни четная, ни нечетная;
2. y убывает, если $x \in (-\infty; 0) \cup (0; +\infty)$;

$$5) y = \frac{1}{(3-x)^2} + 1;$$



- а) y возрастает, если $x < 3$;
 y убывает, если $x > 3$;
- б) y – ни четная, ни нечетная;

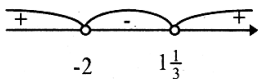
220.

$$1) (3x+1)^4 > 625;$$

$$(3x+1)^2 - 25 > 0, \text{ т.к. } (3x+1)^2 + 25 > 0;$$

$$(3x+1-5)(3x+1+5) > 0;$$

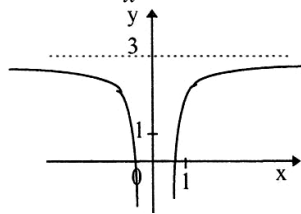
получим $(3x-4)(3x+6) > 0$.



Значит, $x < -2$ или $x > 1\frac{1}{3}$.

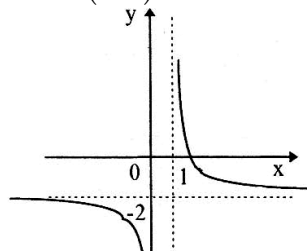
Ответ: $x \in (-\infty; -2) \cup (1\frac{1}{3}; +\infty)$.

$$4) y = 3 - \frac{1}{x^2};$$



1. y – четная;
2. y возрастает, если $x > 0$
 y убывает, если $x < 0$;

$$6) y = \frac{1}{(x-1)^3} - 2;$$



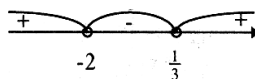
- а) y убывает, если $x < 1$,
и $x > 1$;
- б) y – ни четная, ни нечетная.

$$2) (3x^2 + 5x)^5 \leq 32;$$

$$(3x^2 + 5x) \leq 2.$$

Тогда $3x^2 + 5x - 2 \leq 0$;

$$x_1 = -2; \quad x_2 = \frac{1}{3}$$



Поэтому $-2 \leq x \leq 1\frac{1}{3}$;

$$(x+2)(x - \frac{1}{3}) \leq 0.$$

Ответ: $x \in [-2; 1\frac{1}{3}]$.

221.

1) $\sqrt{2x^2 + 5x - 3} = x + 1$ по О.Д.З.

$$\begin{cases} x + 1 \geq 0 \\ 2x^2 + 5x - 3 \geq 0 \end{cases}; x \in \left(\frac{1}{2}; +\infty\right).$$

Возводим в квадрат

$$2x^2 + 5x - 3 = x^2 + 2x + 1;$$

$$x^2 + 3x - 4 = 0. \text{ Решим:}$$

$$x_1 = 1; x_2 = -4 - \text{ не входит в О.Д.З.}$$

Ответ: $x = 1$.

2) $\sqrt{3x^2 - 4x + 2} = x + 4$; О.Д.З.:

$$\begin{cases} x + 4 \geq 0 \\ 3x^2 - 4x + 2 \geq 0 \end{cases}; x \in (-4; +\infty).$$

Возводим в квадрат

$$3x^2 - 4x + 2 = x^2 + 8x + 16;$$

$$2x^2 - 12x - 14 = 0;$$

$$x^2 - 6x - 7 = 0. \text{ Решим:}$$

$$x_1 = 7; x_2 = -1 \text{ входят в О.Д.З.}$$

Ответ: $x_1 = 7; x_2 = -1$.

3) $\sqrt{x+11} = 1 + \sqrt{x}$; О.Д.З.: $\begin{cases} x+11 \geq 0 \\ x \geq 0 \end{cases}; x \geq 0$.

Возводим в квадрат

$$x + 11 = 1 + 2\sqrt{x} + x;$$

$$10 = 2\sqrt{x};$$

$$\sqrt{x} = 5.$$

Тогда $x = 25$ входит в О.Д.З.

Ответ: $x = 25$.

4) $\sqrt{x+19} = 1 + \sqrt{x}$; О.Д.З.: $\begin{cases} x+19 \geq 0 \\ x \geq 0 \end{cases}; x \geq 0$.

Возводим в квадрат

$$x + 19 = 1 + 2\sqrt{x} + x;$$

$$2\sqrt{x} = 18;$$

$$\sqrt{x} = 9;$$

$x = 81$ входит в О.Д.З.

Ответ: $x = 81$.

$$5) \sqrt{x+3} + \sqrt{2x-3} = 6; \quad \text{О.Д.З.:} \begin{cases} x+3 \geq 0 \\ 2x-3 \geq 0 \end{cases}; \quad x \in [1,5; \infty)$$

$$\sqrt{2x-3} = 6 - \sqrt{x+3}.$$

Возводим в квадрат

$$2x - 3 = 36 - 12\sqrt{x+3} + x + 3;$$

$$x - 6 - 36 = -12\sqrt{x+3}.$$

Возводим в квадрат

$$(x - 42) = -12\sqrt{x+3}, \quad \text{О.Д.З.} \quad x - 42 \leq 0, \quad \text{т.е.} \quad x \in [1,5; 42];$$

$$(x^2 - 84x + 1764) = 144(x + 3);$$

$$x^2 - 228x + 1332 = 0. \quad \text{Решим}$$

$$x_1 = 222; x_2 = 6, \quad x_1 = 222 - \text{не входит в О.Д.З.}$$

Ответ: $x = 6$.

$$6) \sqrt{7-x} + \sqrt{3x-5} = 4; \quad \text{О.Д.З.:} \begin{cases} 7-x \geq 0 \\ 3x-5 \geq 0 \end{cases}; \quad x \in \left[\frac{5}{3}; 7\right];$$

$$\sqrt{3x-5} = 4 - \sqrt{7-x}.$$

Возводим в квадрат

$$3x - 5 = 16 - 8\sqrt{7-x} + 7 - x;$$

$$4x - 5 - 16 - 7 = -8\sqrt{7-x};$$

$$4x - 28 = -8\sqrt{7-x};$$

$$x - 7 = -2\sqrt{7-x}; \quad \text{О.Д.З.:}$$

$$x - 7 \leq 0, \quad \text{т.е.} \quad x \in \left[\frac{5}{3}; 7\right].$$

$$\text{Возводим в квадрат } x^2 - 14x + 49 = 28 - 4x;$$

$$x^2 - 10x + 21 = 0. \quad \text{Решим } x_1 = 3; x_2 = 7 \text{ входят в О.Д.З.}$$

Ответ: $x_1 = 3; x_2 = 7$.

222.

$$1) \sqrt{x^2 - 8x} > 3; \quad x > 9 \text{ или } x < -1;$$

$$\begin{cases} x^2 - 8x \geq 0 \\ x^2 - 8x > 9 \end{cases} \begin{cases} x(x-8) \geq 0 \\ x^2 - 8x - 9 > 0 \end{cases}$$



Ответ: $x \in (-\infty; -1) \cup (9; +\infty)$.

$$2) \sqrt{x^2 - 3x} < 2;$$

$$\begin{cases} x^2 - 3x \geq 0 \\ x^2 - 3x < 4 \end{cases} \begin{cases} x(x-3) \geq 0 \\ x^2 - 3x - 4 < 0 \end{cases} \begin{cases} x \geq 3 \text{ или } x \leq 0 \\ -1 < x < 4 \end{cases}$$



Ответ: $x \in (-1; 0] \cup [3; 4)$.

$$3) \sqrt{3x-2} > x-2;$$

$$\begin{cases} 3x-2 \geq 0 \\ 3x-2 > x^2-4x+4 \end{cases} \begin{cases} x \geq \frac{2}{3} \\ x^2-7x+6 < 0 \end{cases} \begin{cases} x \geq \frac{2}{3} \\ 1 < x < 6 \end{cases}$$



Ответ: $x \in (1; 6)$.

$$4) \sqrt{2x+1} \leq x-1;$$

$$\begin{cases} 2x+1 \geq 0 \\ x-1 \geq 0 \\ 2x+1 \leq x^2-2x+1 \end{cases} \begin{cases} x \geq -\frac{1}{2} \\ x \geq 1 \\ x^2-4x \geq 0 \end{cases} \begin{cases} x > 1 \\ x \leq 0 \text{ или } x \geq 4 \end{cases}$$



Ответ: $x \in [4; +\infty)$.

Глава IV. Элементы тригонометрии

223.

$$1) 40^\circ = \frac{40\pi}{180} = \frac{2\pi}{9} \text{ рад.};$$

$$2) 120^\circ = \frac{120\pi}{180} = \frac{2\pi}{3} \text{ рад.};$$

$$3) 105^\circ = \frac{105}{180}\pi = \frac{7\pi}{12} \text{ рад.};$$

$$4) 150^\circ = \frac{150}{180}\pi = \frac{5\pi}{6} \text{ рад.};$$

$$5) 75^\circ = \frac{75}{180}\pi = \frac{5\pi}{12} \text{ рад.};$$

$$6) 32^\circ = \frac{32}{180}\pi = \frac{8\pi}{45} \text{ рад.};$$

$$7) 100^\circ = \frac{100}{180}\pi = \frac{5\pi}{9} \text{ рад.};$$

$$8) 140^\circ = \frac{140}{180}\pi = \frac{7\pi}{9} \text{ рад.}$$

224.

$$1) \frac{\pi}{6} = \frac{180^\circ}{6} = 30^\circ;$$

$$2) \frac{\pi}{9} = \frac{180^\circ}{9} = 20^\circ;$$

$$3) \frac{2\pi}{3} = \frac{2 \cdot 180}{3} = 120^\circ;$$

$$4) \frac{3\pi}{4} = \frac{3 \cdot 180^\circ}{4} = 135^\circ;$$

$$5) 2 = \frac{180^\circ}{\pi} \cdot 2 = \left(\frac{360}{\pi}\right)^\circ;$$

$$6) 4 = 4 \cdot \frac{180^\circ}{\pi} = \left(\frac{720}{\pi}\right)^\circ;$$

$$7) 1,5 = \frac{180^\circ}{\pi} \cdot \frac{3}{2} = \left(\frac{270}{\pi}\right)^\circ;$$

$$8) 0,36 = \frac{180^\circ}{\pi} \cdot \frac{36}{100} = \left(\frac{324}{5\pi}\right)^\circ.$$

225.

$$1) \frac{\pi}{2} \approx \frac{3,141}{2} \approx 1,57;$$

$$2) \frac{3}{2}\pi \approx \frac{3 \cdot 3,141}{2} \approx 4,71;$$

$$3) 2\pi \approx 2 \cdot 3,141 = 6,28;$$

$$4) \frac{2}{3}\pi \approx \frac{2 \cdot 3,141}{3} \approx 2,09.$$

226.

$$1) \frac{\pi}{2} < 2;$$

$$2) 2\pi < 6,7;$$

$$3) \pi < 3\frac{1}{5};$$

$$4) \frac{3}{2}\pi < 4,8;$$

$$5) -\frac{\pi}{2} < -\frac{3}{2};$$

$$6) -\frac{3}{2}\pi < -\sqrt{10}.$$

227.

$$а) 60^\circ = \frac{\pi}{3} \text{ рад.};$$

$$б) 90^\circ = \frac{\pi}{2} \text{ рад.};$$

$$в) 45^\circ = \frac{\pi}{4} \text{ рад.};$$

$$г) 120^\circ = \frac{2\pi}{3} \text{ рад.}$$

228.

$$\ell = \alpha R,$$

$$\text{если } \begin{cases} \ell = 0,36 \text{ м} \\ \alpha = 0,9 \end{cases}, \text{ то } R = \frac{\ell}{\alpha} = \frac{0,36}{0,9} = 0,4 \text{ (м).}$$

229.

$$\ell = \alpha R,$$

если $\begin{cases} \ell = 3 \text{ см} \\ R = 1,5 \text{ см} \end{cases}$, то $\alpha = \frac{\ell}{R} = \frac{3}{1,5} = 2$ (рад).

230.

$$S = \frac{R^2}{2} \alpha,$$

если $\alpha = \frac{3\pi}{4}$ и $R = 1$ см, тогда $S = \frac{3\pi}{2 \cdot 4} = \frac{3\pi}{8}$ (см²).

231.

$$S = \frac{R^2}{2} \alpha,$$

если $\begin{cases} R = 2,5 \text{ см} \\ S = 6,25 \text{ см}^2 \end{cases}$, тогда $\alpha = \frac{2S}{R^2} = \frac{2 \cdot 6,25}{6,25} = 2$ (рад).

Ответ: $\alpha = 2$ (рад).

234.

1) Получим $M(0; 1)$.

2) Получим $M(-1; 0)$.

3) Получим $M(-1; 0)$.

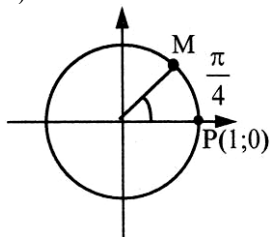
4) Получим $M(0; -1)$.

5) Получим $M(0; -1)$.

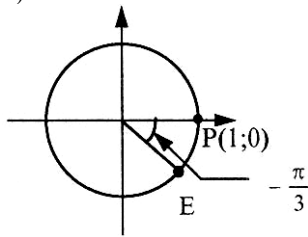
6) Получим $M(1; 0)$.

235.

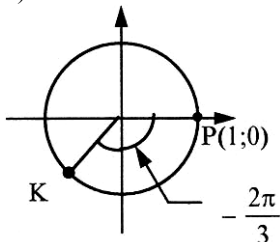
1)



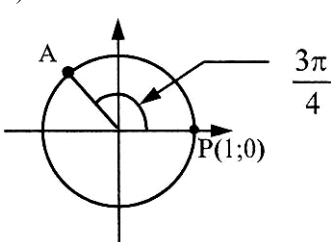
2)



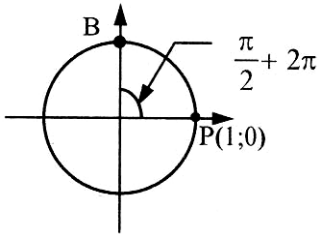
3)



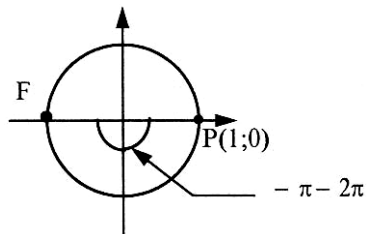
4)



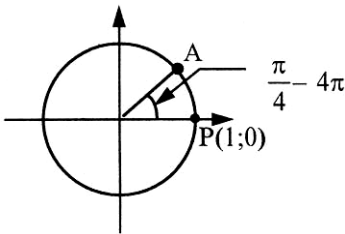
5)



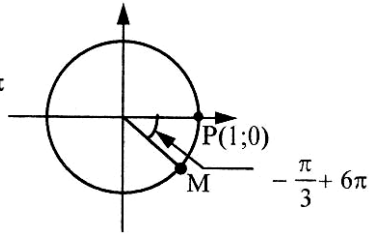
6)



7)



8)



236.

1) I четв.

3) IV четв.

5) I четв.

2) II четв.

4) IV четв.

6) II четв.

237.

1) $A(-1; 0)$;

3) $C(0; 1)$;

5) $E(-1; 0)$;

2) $B(0; 1)$;

4) $D(-1; 0)$;

6) $F(0; 1)$.

238.

1) $\alpha = \pi + 2\pi n, n \in \mathbb{Z}$;

3) $\alpha = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$;

2) $\alpha = 2\pi n, n \in \mathbb{Z}$;

4) $\alpha = -\frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$.

239.

1) $\alpha = 1 \text{ рад.} \approx 57^\circ$, I четв.

2) $\alpha = 2,75 \text{ рад.} \approx 132^\circ$, II четв.

3) $\alpha = 3,16 \text{ рад.} \approx 181^\circ$, III четв.

4) $\alpha = 4,95 \text{ рад.} \approx 282^\circ$, IV четв.

240.

1) $a = 6,7\pi$, $6\frac{7}{10}\pi = \frac{7}{10}\pi + 6\pi$. Тогда $x = \frac{7}{10}\pi$, $n = 3$.

2) $a = 9,8\pi$, $9\frac{4}{5}\pi = 1\frac{4}{5}\pi + 8\pi$. Тогда $x = 1\frac{4}{5}\pi$, $n = 4$.

3) $a = 4\frac{1}{2}\pi$, $4\frac{1}{2}\pi = \frac{\pi}{2} + 4\pi$. Тогда $x = \frac{\pi}{2}$, $n = 2$.

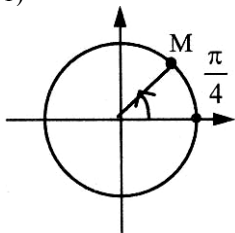
4) $a = 7\frac{1}{3}\pi$, $7\frac{1}{3}\pi = 1\frac{1}{3}\pi + 6\pi$. Тогда $x = 1\frac{1}{3}\pi$, $n = 3$.

5) $a = \frac{11}{2}\pi$, $5\frac{1}{2}\pi = 1\frac{1}{2}\pi + 4\pi$. Тогда $x = 1\frac{1}{2}\pi$, $n = 2$.

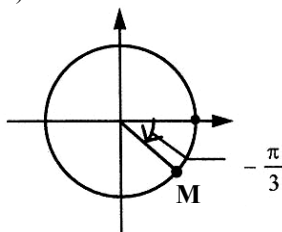
6) $a = \frac{17}{3}\pi$, $5\frac{2}{3}\pi = 1\frac{2}{3}\pi + 4\pi$. Тогда $x = 1\frac{2}{3}\pi$, $n = 2$.

241.

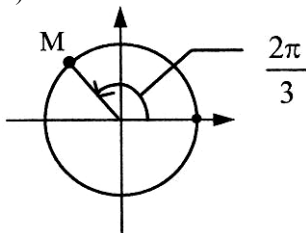
1)



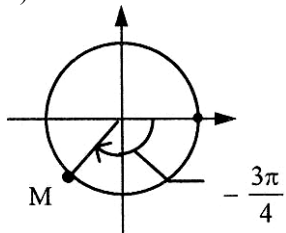
2)



3)

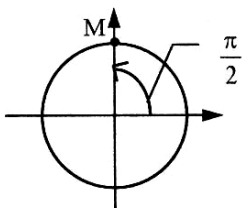


4)

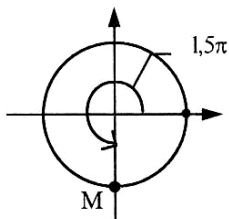
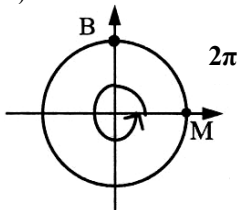


5)

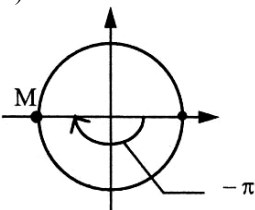
6)



7)



8)



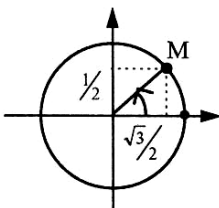
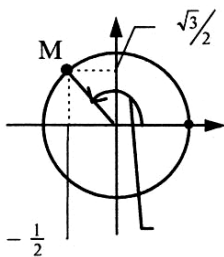
242.

1) $A(0; 1)$; 2) $B(0; 1)$; 3) $C(0; -1)$; 4) $D(0; -1)$.

243.

1) $\alpha = \frac{2\pi}{3} + 2\pi n; n \in \mathbb{Z}$;

2) $\alpha = \frac{\pi}{6} + 2\pi n; n \in \mathbb{Z}$;

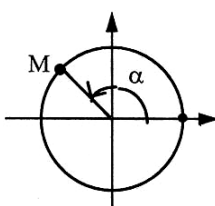
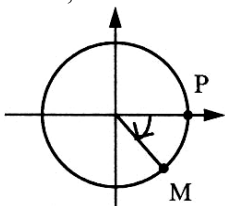


3) $\alpha = -\frac{\pi}{4} + 2\pi n, \alpha = \frac{7\pi}{4} + 2\pi n;$

4) $\alpha = \frac{3\pi}{4} + 2\pi n;$

$n \in \mathbb{Z}$;

$n \in \mathbb{Z}$.



244.

1) $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$;

3) $\operatorname{tg} \frac{5\pi}{6} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$;

5) $\cos(-180^\circ) = -1$;

7) $\cos(-135^\circ) = -\frac{\sqrt{2}}{2}$;

2) $\cos \frac{2\pi}{3} = -\frac{1}{2}$;

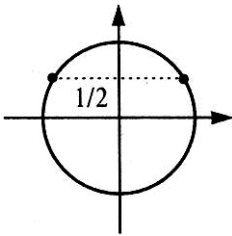
4) $\sin(-90^\circ) = -1$;

6) $\operatorname{tg}\left(-\frac{\pi}{4}\right) = -1$;

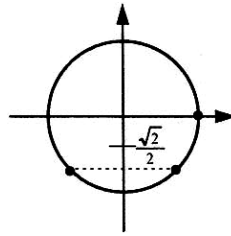
8) $\sin\left(-\frac{5\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

245.

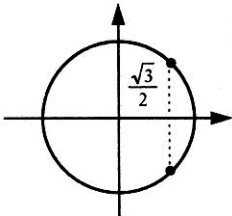
1) $\sin \alpha = \frac{1}{2}$;



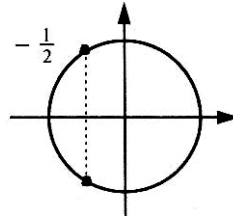
2) $\sin \alpha = -\frac{\sqrt{2}}{2}$;



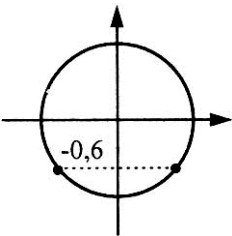
3) $\cos \alpha = \frac{\sqrt{3}}{2}$;



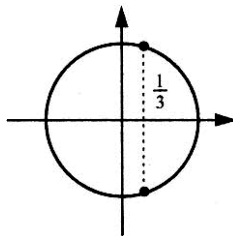
4) $\cos \alpha = -\frac{1}{2}$;



5) $\sin \alpha = -0,6$;



6) $\cos \alpha = \frac{1}{3}$.



246.

1) $\sin \frac{\pi}{2} + \sin \frac{3\pi}{2} = 1 + (-1) = 0$;

2) $\sin\left(-\frac{\pi}{2}\right) + \cos \frac{\pi}{2} = -1 + 0 = -1$;

3) $\sin \pi - \cos \pi = 0 - (-1) = 1$;

4) $\sin 0 - \cos 2\pi = 0 - 1 = -1$;

5) $\sin \pi + \sin 1,5\pi = 0 + (-1) = -1$;

6) $\cos 0 - \cos \frac{3}{2}\pi = 1 - 0 = 1$.

247.

1) $\operatorname{tg} \pi + \cos \pi = 0 - 1 = -1$;

2) $\operatorname{tg} 0^\circ - \operatorname{tg} 180^\circ = 0$;

3) $\operatorname{tg} \pi + \sin \pi = 0$;

4) $\cos \pi - \operatorname{tg} 2\pi = -1 - 0 = -1$.

248.

1) $3\sin \frac{\pi}{6} + 2\cos \frac{\pi}{6} - \operatorname{tg} \frac{\pi}{3} = 3 \cdot \frac{1}{2} + 2 \cdot \frac{\sqrt{3}}{2} - \sqrt{3} = \frac{3}{2}$;

2) $5\sin \frac{\pi}{6} + 3\operatorname{tg} \frac{\pi}{4} - \cos \frac{\pi}{4} - 10\operatorname{tg} \frac{\pi}{4} = 5 \cdot \frac{1}{2} + 3 - \frac{\sqrt{2}}{2} - 10 = \frac{-\sqrt{2}}{2} - 4,5$;

3) $\left(2\operatorname{tg} \frac{\pi}{6} - \operatorname{tg} \frac{\pi}{3}\right) : \cos \frac{\pi}{6} = \left(2 \cdot \frac{1}{\sqrt{3}} - \sqrt{3}\right) : \frac{\sqrt{3}}{2} = \left(\frac{2-3}{3}\right) \cdot \frac{2}{\sqrt{3}} = -\frac{2}{3}$;

4) $\sin \frac{\pi}{3} \cdot \cos \frac{\pi}{6} - \operatorname{tg} \frac{\pi}{4} = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - 1 = \frac{3}{4} - 1 = -\frac{1}{4}$.

249.

1) $2 \sin x = 0$.

2) $\frac{1}{2} \cos x = 0$.

Тогда $\sin x = 0$;

Значит, $\cos x = 0$;

$x = \pi n, n \in \wedge$;

$x = \frac{\pi}{2} + \pi n, n \in \wedge$;

3) $\cos x - 1 = 0$.

4) $1 - \sin x = 0$.

Поэтому $\cos x = 1$;

Тогда $\sin x = 1$;

$x = 2\pi n, n \in \wedge$;

$x = \frac{\pi}{2} + 2\pi n, n \in \wedge$.

250.

1) да, т.к. $-1 < 0,49 < 1$;

2) да, т.к. $1 > -0,875 > -1$;

3) нет, т.к. $-\sqrt{2} < -1$;

4) да, т.к. $-1 < 2 - \sqrt{2} < 1$.

251.

$$1) \underline{2 \sin \alpha + \sqrt{2} \cos \alpha} = 2 \sin \frac{\pi}{4} + \sqrt{2} \sin \frac{\pi}{4} = 2 \cdot \frac{\sqrt{2}}{2} + \sqrt{2} \cdot \frac{\sqrt{2}}{2} = \sqrt{2} + 1$$

$$2) \underline{0,5 \cos \alpha - \sqrt{3} \sin \alpha} =$$

$$= 0,5 \cos \frac{\pi}{3} - \sqrt{3} \sin \frac{\pi}{3} = \frac{1}{2} \cdot \frac{1}{2} - \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{1}{4} - \frac{3}{2} = -\frac{5}{4}$$

$$3) \underline{\sin 3\alpha - \cos 2\alpha} = \sin \frac{3\pi}{6} - \cos \frac{2\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$4) \underline{\cos \frac{\alpha}{2} + \sin \frac{\alpha}{3}} = \cos \frac{\pi}{4} + \sin \frac{\pi}{6} = \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{\sqrt{2} + 1}{2}$$

252.

$$1) \sin x = -1$$

$$2) \cos x = -1$$

$$x = -\frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$$

$$x = \pi + 2\pi n, n \in \mathbb{Z}$$

$$3) \sin 3x = 0$$

$$4) \cos 0,5x = 0$$

$$\text{Тогда } 3x = \pi n, n \in \mathbb{Z}$$

$$\text{Значит } 0,5x = \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$$

$$x = \frac{\pi n}{3}, n \in \mathbb{Z}$$

$$x = \pi + 2\pi n, n \in \mathbb{Z}$$

$$5) \cos 2x - 1 = 0$$

$$6) 1 - \cos 3x = 0$$

$$\cos 2x = 1$$

$$\cos 3x = 1$$

$$\text{Отсюда } 2x = 2\pi n, n \in \mathbb{Z}$$

$$3x = 2\pi n, n \in \mathbb{Z}$$

$$x = \pi n, n \in \mathbb{Z}$$

$$x = \frac{2\pi n}{3}, n \in \mathbb{Z}$$

253.

$$1) \cos 12^\circ \approx 0,98; 2) \sin 38^\circ \approx 0,62$$

$$3) \operatorname{tg} 100^\circ \approx -5,67$$

$$4) \sin 400^\circ = \sin(360^\circ + 40^\circ) = \sin 40^\circ \approx 0,64$$

$$5) \cos 2,7 \approx \cos 158^\circ = \cos(180^\circ - 22^\circ) = -\cos 22^\circ \approx -0,93$$

$$6) \operatorname{tg}(-13) \approx -\operatorname{tg} 745^\circ = -\operatorname{tg}(720^\circ + 25^\circ) = -\operatorname{tg}(360^\circ \cdot 2 + 25^\circ) = -\operatorname{tg} 25^\circ \approx -0,47$$

$$7) \sin \frac{\pi}{6} = 0,5$$

$$8) \cos \left(-\frac{\pi}{7} \right) \approx \cos 26^\circ \approx 0,9$$

254.

- 1) I четв.
- 2) II четв.
- 3) III четв.
- 4) II четв.
- 5) I четв.
- 6) II четв.

255.

- 1) $\sin \frac{5\pi}{4} < 0$, т.к. $\pi < \frac{5\pi}{4} < \frac{3\pi}{2}$ III четв.
- 2) $\sin \frac{5\pi}{6} > 0$, т.к. $\frac{\pi}{2} < \frac{5\pi}{6} < \pi$ II четв.
- 3) $\sin(-\frac{5\pi}{8}) < 0$, т.к. $-\pi < -\frac{5\pi}{8} < -\frac{\pi}{2}$ IV четв.
- 4) $\sin(-\frac{4\pi}{3}) > 0$, т.к. $-\frac{3\pi}{2} < -\frac{4\pi}{3} < -\pi$ II четв.
- 5) $\sin 740^\circ > 0$, I четв.
- 6) $\sin 510^\circ > 0$, II четв.

256.

- 1) $\cos \frac{2\pi}{3} < 0$, II четв. 2) $\cos \frac{7\pi}{6} < 0$, III четв.
- 3) $\cos(-\frac{3\pi}{4}) < 0$, III четв. 4) $\cos(-\frac{2\pi}{5}) > 0$, IV четв.
- 5) $\cos 290^\circ > 0$, IV четв. 6) $\cos(-150^\circ) < 0$, III четв.

257.

- 1) $\operatorname{tg} \frac{5}{6}\pi < 0$
- 2) $\operatorname{tg} \frac{12}{5}\pi > 0$
- $\operatorname{ctg} \frac{5}{6}\pi < 0$, II четв.
- $\operatorname{ctg} \frac{12}{5}\pi > 0$, II четв.
- 3) $\operatorname{tg}\left(\frac{-3\pi}{5}\right) > 0$
- 4) $\operatorname{tg}\left(-\frac{5\pi}{4}\right) < 0$
- $\operatorname{ctg}\left(\frac{-3\pi}{5}\right) > 0$, III четв.
- $\operatorname{ctg}\left(-\frac{5\pi}{4}\right) < 0$, II четв.
- 5) $\operatorname{tg} 190^\circ > 0$
- 6) $\operatorname{tg} 283^\circ < 0$
- $\operatorname{ctg} 190^\circ > 0$, III четв.
- $\operatorname{ctg} 283^\circ < 0$, IV четв.
- 7) $\operatorname{tg} 172^\circ < 0$
- 8) $\operatorname{tg} 200^\circ > 0$
- $\operatorname{ctg} 172^\circ < 0$, II четв.
- $\operatorname{ctg} 200^\circ > 0$, III четв.

258.

1) если $\pi < \alpha < \frac{3\pi}{2}$, то

$$\sin\alpha < 0, \cos\alpha < 0, \operatorname{tg}\alpha > 0, \operatorname{ctg}\alpha > 0$$

2) если $\frac{3\pi}{2} < \alpha < \frac{7\pi}{4}$, то

$$\sin\alpha < 0, \cos\alpha > 0, \operatorname{tg}\alpha < 0, \operatorname{ctg}\alpha < 0$$

3) если $\frac{7\pi}{4} < \alpha < 2\pi$, то

$$\sin\alpha < 0, \cos\alpha > 0, \operatorname{tg}\alpha < 0, \operatorname{ctg}\alpha < 0$$

4) если $2\pi < \alpha < 2,5\pi$, то

$$\sin\alpha > 0, \cos\alpha > 0, \operatorname{tg}\alpha > 0, \operatorname{ctg}\alpha > 0$$

259.

a) $\sin 1 > 0, \cos 1 > 0, \operatorname{tg} 1 > 0$

б) $\sin 3 > 0, \cos 3 < 0, \operatorname{tg} 3 < 0$

в) $\sin(-3,4) > 0, \cos(-3,4) < 0,$

$\operatorname{tg}(-3,4) < 0$

г) $\sin(-1,3) < 0, \cos(-1,3) > 0,$

$\operatorname{tg}(-1,3) < 0$

260.

1) $\sin\left(\frac{\pi}{2} - \alpha\right) > 0$ 2) $\cos\left(\frac{\pi}{2} + \alpha\right) < 0$ 3) $\operatorname{tg}\left(\frac{3\pi}{2} - \alpha\right) > 0$

4) $\sin(\pi - \alpha) > 0$ 5) $\cos(\alpha - \pi) < 0$ 6) $\operatorname{tg}(\alpha - \pi) > 0$

7) $\cos\left(\alpha - \frac{\pi}{2}\right) > 0$ 8) $\operatorname{ctg}\left(\alpha - \frac{\pi}{2}\right) < 0$

261.

1) если $0 < \alpha < \frac{\pi}{2}$ и

2) если $\frac{\pi}{2} < \alpha < \pi$ и

$\pi < \alpha < \frac{3\pi}{2}$, то – знаки синуса

$\frac{3\pi}{2} < \alpha < 2\pi$, то – знаки синуса

и косинуса совпадают.

и косинуса различны.

262.

1) $\sin\frac{2\pi}{3} \cdot \sin\frac{3\pi}{4} > 0$

2) $\cos\frac{2\pi}{3} \cdot \cos\frac{\pi}{6} < 0$

т.к. $\sin\frac{2\pi}{3} > 0$ и $\sin\frac{3\pi}{4} > 0$

т.к. $\cos\frac{2\pi}{3} < 0, \cos\frac{\pi}{6} > 0$

$$3) \frac{\sin \frac{2\pi}{3}}{\cos \frac{3\pi}{4}} < 0,$$

т.к. $\sin \frac{2\pi}{3} > 0$ и $\cos \frac{3\pi}{4} < 0$;

$$4) \operatorname{tg} \frac{5\pi}{4} + \sin \frac{\pi}{4} > 0,$$

т.к. $\operatorname{tg} \frac{5\pi}{4}$ и $\sin \frac{\pi}{4} > 0$.

263.

1) $\sin 0,7 > \sin 4$,

т.к. $\sin 0,7 > 0$, $\sin 4 < 0$;

2) $\cos 1,3 > \cos 2,3$,

т.к. $\cos 1,3 > 0$, $\cos 2,3 < 0$.

264.

1) $\sin(5\pi + x) = 1$;

$\sin(4\pi + \pi + x) = 1$, но

$\sin(\alpha + 2k\pi) = \sin \alpha$, где $k \in \wedge$

тогда $\sin(\pi + x) = 1$;

$$\pi + x = \frac{\pi}{2} + 2\pi n,$$

$$\text{и } x = -\frac{\pi}{2} + 2\pi n, n \in \wedge;$$

2) $\cos(x + 3\pi) = 0$;

$\cos(x + \pi + 2\pi) = 0$, но т.к.

$\cos(2\pi k + \alpha) = \cos \alpha$, то

$\cos(x + \pi) = 0$;

$$n \in \mathbb{Z} \quad x + \pi = \frac{\pi}{2} + \pi n, n \in \wedge$$

$$x = \frac{\pi}{2} + \pi n, n \in \wedge;$$

$$3) \cos\left(\frac{5\pi}{2} + x\right) = -1;$$

$$\cos\left(2\pi + \frac{\pi}{2} + x\right) = -1,$$

т.к. $\cos(\alpha + 2\pi k) = \cos \alpha$, то

$$\cos\left(\frac{\pi}{2} + x\right) = -1;$$

$$\frac{\pi}{2} + x = \pi + 2\pi n$$

$$\text{и } x = \frac{\pi}{2} + 2\pi n,$$

$n \in \wedge$;

$$4) \sin\left(\frac{9}{2}\pi + x\right) = -1;$$

$$\sin\left(2 \cdot 2\pi + \frac{\pi}{2} + x\right) = -1,$$

т.к. $\sin(2\pi k + \alpha) = \sin \alpha$, то

$$\sin\left(\frac{\pi}{2} + x\right) = -1;$$

$$\frac{\pi}{2} + x = -\frac{\pi}{2} + 2\pi n$$

$$\text{и } x = \pi + 2\pi n,$$

$n \in \wedge$.

265.

т.к. $\sin \alpha + \cos \alpha < 0$, то $M \in$ III четв., где $\cos \alpha < 0$, $\sin \alpha < 0$.

т.к. $\sin \alpha - \cos \alpha > 1$, то $\sin \alpha > 0$, $\cos \alpha < 0$, значит, $M \in$ II четв.

267.

1) Т.к. $\frac{3\pi}{2} < \alpha < 2\pi$, то $\sin \alpha < 0$, тогда

$$\sin \alpha = -\sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \sqrt{\frac{12^2}{13^2}} = -\frac{12}{13};$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{-12 \cdot 13}{13 \cdot 5} = -\frac{12}{5}.$$

2) Т.к. $\frac{\pi}{2} < \alpha < \pi$,

то $\cos \alpha < 0$, тогда

$$\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - 0,64} = -\sqrt{0,36} = -0,6;$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{0,8}{-0,6} = -\frac{4}{3}.$$

3) Т.к. $\frac{\pi}{2} < \alpha < \pi$, то $\sin \alpha > 0$, поэтому

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \sqrt{\frac{4^2}{5^2}} = \frac{4}{5};$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = -\frac{4}{5} \cdot \frac{5}{3} = -\frac{4}{3};$$

$$\operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha} = -\frac{3}{4}.$$

4) Т.к. $\pi < \alpha < \frac{3\pi}{2}$, то $\cos \alpha < 0$, тогда

$$\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \frac{4}{25}} = -\sqrt{\frac{21}{25}} = -\frac{\sqrt{21}}{5};$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{2}{5}}{\frac{\sqrt{21}}{5}} = \frac{2}{\sqrt{21}};$$

$$\operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha} = \frac{\sqrt{21}}{2}.$$

5) Т.к. $\pi < \alpha < \frac{3\pi}{2}$,

то $\sin \alpha < 0$ и $\cos \alpha < 0$;

$$\cos^2 \alpha = \frac{1}{1 + \operatorname{tg}^2 \alpha};$$

$$\sin \alpha = -\sqrt{1 - \cos^2 \alpha};$$

$$\cos \alpha = -\frac{1}{\sqrt{1+\operatorname{tg}^2 \alpha}};$$

$$\sin \alpha = -\sqrt{1-\frac{64}{289}};$$

$$\cos \alpha = -\sqrt{\frac{64}{289}};$$

$$\sin \alpha = -\sqrt{\frac{225}{289}};$$

$$\cos \alpha = -\frac{8}{17};$$

$$\sin \alpha = -\frac{15}{17}.$$

6) Т.к. $\frac{3\pi}{2} < \alpha < 2\pi$, то $\sin \alpha < 0$, а $\cos \alpha > 0$

$$\sin^2 \alpha = \frac{1}{1+\operatorname{ctg}^2 \alpha};$$

$$\cos \alpha = \sqrt{1-\sin^2 \alpha};$$

$$\sin \alpha = \frac{-1}{\sqrt{1+\operatorname{ctg}^2 \alpha}};$$

$$\cos \alpha = \sqrt{1-\frac{1}{10}};$$

$$\sin \alpha = -\sqrt{\frac{1}{10}}; \sin \alpha = -\frac{1}{\sqrt{10}};$$

$$\cos \alpha = \frac{3}{\sqrt{10}}.$$

268.

1) если $\begin{cases} \sin \alpha = 1 \\ \cos \alpha = 1 \end{cases}$,

$1 + 1 = 2 \neq 1$, нет;

2) если $\begin{cases} \sin \alpha = -\frac{4}{5} \\ \cos \alpha = -\frac{3}{5} \end{cases}$,

$$\frac{16}{25} + \frac{9}{25} = 1, \text{ да};$$

3) если $\begin{cases} \sin \alpha = 0 \\ \cos \alpha = -1 \end{cases}$,

$0 + 1 = 1$, да;

4) если $\begin{cases} \sin \alpha = \frac{1}{3} \\ \cos \alpha = -\frac{1}{2} \end{cases}$,

$$\frac{1}{9} + \frac{1}{4} = \frac{13}{36} \neq 1, \text{ нет}.$$

269.

$$1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}; \quad 1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha};$$

$$1) \begin{cases} \sin \alpha = \frac{1}{5}; \\ \operatorname{tg} \alpha = \frac{1}{\sqrt{24}} \end{cases}; \quad \begin{cases} \sin \alpha = \frac{1}{5}; \\ \operatorname{ctg}^2 \alpha = 24 \end{cases};$$

$$1 + 24 = \frac{1}{\left(\frac{1}{5}\right)^2} = 25.$$

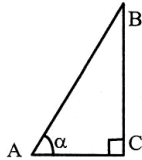
Ответ: да.

$$2) \begin{cases} \cos \alpha = \frac{3}{4}; \\ \operatorname{ctg} \alpha = \frac{\sqrt{7}}{3} \end{cases}; \quad \begin{cases} \cos \alpha = \frac{3}{4}; \\ \operatorname{tg}^2 \alpha = \frac{9}{7} \end{cases};$$

$$1 + \frac{9}{7} = \frac{1}{\left(\frac{3}{4}\right)^2}, \quad \frac{16}{7} \neq \frac{16}{9}.$$

Ответ: нет.

270.



Пусть: $\angle C = 90^\circ$;

$\angle A = \alpha$;

$$\sin \alpha = \frac{2\sqrt{10}}{11};$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha};$$

$$\cos \alpha = \sqrt{1 - \frac{40}{121}} = \sqrt{\frac{81}{121}};$$

$$\cos \alpha = \frac{9}{11};$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha};$$

$$\operatorname{tg} \alpha = \frac{2\sqrt{10}}{11} : \frac{9}{11};$$

$$\operatorname{tg} \alpha = \frac{2\sqrt{10}}{9}.$$

271.

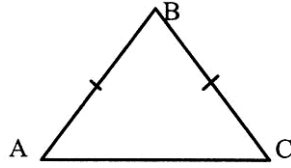
Пусть $AB = BC$,

$$\operatorname{tg} \angle B = 2\sqrt{2};$$

$$\cos^2 \alpha = \frac{1}{1 + \operatorname{tg}^2 \alpha};$$

$$\cos^2 \alpha = \frac{1}{9}. \text{ Т.к. } 0 < \angle B < 90^\circ, \text{ то}$$

$$\cos \alpha = \frac{1}{3}.$$



272.

$$\cos^4 \alpha - \sin^4 \alpha = \frac{1}{8};$$

$$(\cos^2 \alpha - \sin^2 \alpha)(\cos^2 \alpha + \sin^2 \alpha) = (-\cos^2 \alpha - \sin^2 \alpha) = \frac{1}{8}.$$

$$\text{Т.к. } \sin^2 \alpha = 1 - \cos^2 \alpha, \text{ то } \cos^2 \alpha - (1 - \cos^2 \alpha) = \frac{1}{8};$$

$$2 \cos^2 \alpha = \frac{9}{8} \quad \cos^2 \alpha = \frac{9}{16}, \quad \cos \alpha = \pm \frac{3}{4}.$$

$$\text{Ответ: } \cos \alpha = \pm \frac{3}{4}.$$

273.

$$1) \sin \alpha = \frac{2\sqrt{3}}{5};$$

$$2) \cos \alpha = -\frac{1}{\sqrt{5}};$$

$$\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha};$$

$$\sin \alpha = \pm \sqrt{1 - \cos^2 \alpha};$$

$$\cos \alpha = \pm \sqrt{1 - \frac{12}{25}};$$

$$\sin \alpha = \pm \sqrt{1 - \frac{1}{5}};$$

$$\cos \alpha = \pm \frac{\sqrt{13}}{5};$$

$$\sin \alpha = \pm \frac{2}{\sqrt{5}}.$$

274.

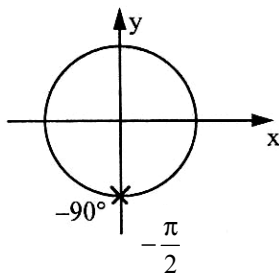
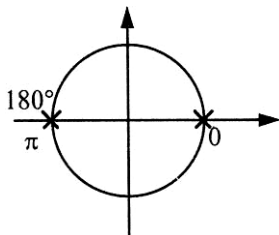
$$\operatorname{tg} \alpha = 2, \text{ значит, } \operatorname{ctg} \alpha = \frac{1}{2};$$

$$1) \frac{\operatorname{ctg} \alpha + \operatorname{tg} \alpha}{\operatorname{ctg} \alpha - \operatorname{tg} \alpha} = \frac{\frac{1}{2} + 2}{\frac{1}{2} - 2} = \frac{2,5}{-1,5} = -\frac{5}{3};$$

$$2) \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} = \frac{\frac{\sin \alpha}{\cos \alpha} - \frac{\cos \alpha}{\cos \alpha}}{\frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\cos \alpha}} = \frac{\operatorname{tg} \alpha - 1}{\operatorname{tg} \alpha + 1} = \frac{2-1}{2+1} = \frac{1}{3};$$

$$3) \frac{2 \sin \alpha + 3 \cos \alpha}{3 \sin \alpha - 5 \cos \alpha} = \frac{2 \operatorname{tg} \alpha + 3}{3 \operatorname{tg} \alpha - 5} = \frac{4+3}{6-5} = 7;$$

$$4) \frac{\sin^2 \alpha + 2 \cos^2 \alpha}{\sin^2 \alpha - \cos^2 \alpha} = \frac{\frac{\sin^2 \alpha}{\cos^2 \alpha} + 2 \frac{\cos^2 \alpha}{\cos^2 \alpha}}{\frac{\sin^2 \alpha}{\cos^2 \alpha} - \frac{\cos^2 \alpha}{\cos^2 \alpha}} = \frac{\operatorname{tg}^2 \alpha + 2}{\operatorname{tg}^2 \alpha - 1} = \frac{4+2}{4-1} = 2.$$



$$\cos^2 x - \cos x + 1 = 0.$$

Пусть $t = \cos x$. Тогда

$t^2 - t + 1 = 0$. Решим уравнение

$D = 1 - 4 < 0$. Решения нет.

$$4) 3 - \cos x = 3 \cos^2 x + 3 \sin^2 x.$$

Т.к. $\sin^2 x + \cos^2 x = 1$, то

$$3 - \cos x = 3;$$

$$\cos x = 0;$$

$$x = \frac{\pi}{2} + \pi n; n \in \mathbb{Z}.$$

276.

$$1) 2 \sin x + \sin^2 x + \cos^2 x = 1,$$

т.к. $\sin^2 x + \cos^2 x = 1$, то

$$2 \sin x + 1 = 1,$$

$$2 \sin x = 0.$$

Тогда $\sin x = 0$

и $x = k\pi, k \in \mathbb{Z}$;

$$2) \sin^2 x - 2 = \sin x - \cos^2 x;$$

$$\sin^2 x + \cos^2 x - 2 = \sin x,$$

т.к. $\sin^2 x + \cos^2 x = 1$, то

$$\sin x = -1,$$

значит, $x = -\frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$;

$$3) 3 \cos^2 x - 1 = \cos x - 2 \sin^2 x;$$

$$3 \cos^2 x + 2 \sin^2 x - 1 = \cos x;$$

$$\cos^2 x + 2 - 1 = \cos x;$$

277.

1) Т.к. $1 - \cos^2 \alpha = \sin^2 \alpha$, то
 $(1 - \cos \alpha)(1 + \cos \alpha) = \sin^2 \alpha$.

3) Т.к. $\operatorname{tg}^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha}$ и
 $\cos^2 \alpha = 1 - \sin^2 \alpha$, то
 $\frac{\sin^2 \alpha}{1 - \sin^2 \alpha} = \operatorname{tg}^2 \alpha$.

5) Т.к. $\cos^2 \alpha + \sin^2 \alpha = 1$ и
 $\cos^2 \alpha = \frac{1}{1 + \operatorname{tg}^2 \alpha}$, то
 $\frac{1}{1 + \operatorname{tg}^2 \alpha} + \sin^2 \alpha = 1$.

2) Т.к. $\sin^2 \alpha + \cos^2 \alpha = 1$, то
 $2 - \sin^2 \alpha - \cos^2 \alpha = 1$.

4) Т.к. $\operatorname{ctg}^2 \alpha = \frac{\cos^2 \alpha}{\sin^2 \alpha}$
и $\sin^2 \alpha = 1 - \cos^2 \alpha$, то
 $\frac{\cos^2 \alpha}{1 - \cos^2 \alpha} = \operatorname{ctg}^2 \alpha$.

6) Т.к. $\sin^2 \alpha + \cos^2 \alpha = 1$
и $\sin^2 \alpha = \frac{1}{1 + \operatorname{ctg}^2 \alpha}$, то
 $\frac{1}{1 + \operatorname{ctg}^2 \alpha} + \cos^2 \alpha = 1$.

278.

$$\cos \alpha \cdot \operatorname{tg} \alpha - 2 \sin \alpha = \sin \alpha - 2 \sin \alpha = -\sin \alpha;$$

$$\cos \alpha - \sin \alpha \cdot \operatorname{ctg} \alpha = \cos \alpha - \cos \alpha = 0;$$

$$\frac{\sin^2 \alpha}{1 + \cos \alpha} = \frac{1 - \cos^2 \alpha}{1 + \cos \alpha} = \frac{(1 + \cos \alpha)(1 - \cos \alpha)}{1 + \cos \alpha} = 1 - \cos \alpha;$$

$$\frac{\cos^2 \alpha}{1 - \sin \alpha} = \frac{1 - \sin^2 \alpha}{1 - \sin \alpha} = \frac{(1 + \sin \alpha)(1 - \sin \alpha)}{1 - \sin \alpha} = 1 + \sin \alpha.$$

279.

1) $\frac{\sin^2 \alpha - 1}{1 - \cos^2 \alpha} = \frac{-\cos^2 \alpha}{\sin^2 \alpha} = -\operatorname{ctg}^2 \alpha$; $\operatorname{ctg} \frac{\pi}{4} = 1$; $-\operatorname{ctg}^2 \frac{\pi}{4} = -1$;

2) $\frac{1}{\cos^2 \alpha} - 1 = \operatorname{tg}^2 \alpha$; $\operatorname{tg} \frac{\pi}{3} = \sqrt{3}$; $\operatorname{ctg}^2 \frac{\pi}{3} = 3$;

3) $\cos^2 \alpha + \operatorname{ctg}^2 \alpha + \sin^2 \alpha = 1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}$,

$$\sin \frac{\pi}{6} = \frac{1}{2}, \quad \frac{1}{\sin^2 \frac{\pi}{6}} = 4;$$

4) $\cos^2 \alpha + \operatorname{tg}^2 \alpha + \sin^2 \alpha = 1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$,

$$\cos \frac{\pi}{3} = \frac{1}{2}, \quad \frac{1}{\cos^2 \frac{\pi}{3}} = 4.$$

280.

$$1) (1 - \sin^2 \alpha)(1 - \operatorname{tg}^2 \alpha) = 1.$$

$$\text{Тогда } (1 - \sin^2 \alpha) \cdot \frac{1}{\cos^2 \alpha} = 1;$$

$$\cos^2 \alpha \cdot \frac{1}{\cos^2 \alpha} = 1, 1 = 1.$$

Получим тождество.

$$2) \sin^2(1 + \operatorname{ctg}^2 \alpha) - \cos^2 \alpha = \sin^2 \alpha.$$

$$\text{Значит } \sin^2 \alpha \cdot \frac{1}{\sin^2 \alpha} - \cos^2 \alpha = \sin^2 \alpha;$$

$$1 - \cos^2 \alpha = \sin^2 \alpha.$$

$$\text{Тождество } \sin^2 \alpha = \sin^2 \alpha.$$

281.

$$1) (1 + \operatorname{tg}^2 \alpha) \cdot \cos^2 \alpha = \frac{1}{\cos^2 \alpha} \cdot \cos^2 \alpha = 1;$$

$$2) \sin^2 \alpha (1 + \operatorname{ctg}^2 \alpha) = \sin^2 \alpha \cdot \frac{1}{\sin^2 \alpha} = 1;$$

$$3) \left(1 + \operatorname{tg}^2 \alpha + \frac{1}{\sin^2 \alpha}\right) \sin^2 \alpha \cdot \cos^2 \alpha = \frac{\sin^2 \alpha + \cos^2 \alpha}{\sin^2 \alpha \cdot \cos^2 \alpha} \cdot \sin^2 \alpha \cdot \cos^2 \alpha = 1;$$

$$4) \frac{1 + \operatorname{tg}^2 \alpha}{1 + \operatorname{ctg}^2 \alpha} - \operatorname{tg}^2 \alpha = \frac{1/\cos^2 \alpha}{1/\sin^2 \alpha} - \operatorname{tg}^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} - \operatorname{tg}^2 \alpha = 0.$$

282.

$$1) (1 - \cos 2\alpha)(1 + \cos 2\alpha) = \sin^2 2\alpha;$$

$$1 - \cos^2 2\alpha = \sin^2 2\alpha;$$

$$\sin^2 2\alpha = \sin^2 2\alpha. \text{ Верное тождество.}$$

$$2) \frac{\sin \alpha - 1}{\cos^2 \alpha} = \frac{-1}{1 + \sin \alpha};$$

$$\frac{\sin \alpha - 1}{1 - \sin^2 \alpha} = \frac{-1}{1 + \sin \alpha};$$

$$\frac{\sin \alpha - 1}{(1 - \sin \alpha)(1 + \sin \alpha)} = -\frac{1}{1 + \sin \alpha};$$

$$\frac{1}{-(1 + \sin \alpha)} = -\frac{1}{1 + \sin \alpha}. \text{ Верно.}$$

$$3) \cos^4 \alpha - \sin^4 \alpha = \cos^2 \alpha - \sin^2 \alpha;$$

$$(\cos^2 \alpha + \sin^2 \alpha)(\cos^2 \alpha - \sin^2 \alpha) = \cos^2 \alpha - \sin^2 \alpha;$$

$$\cos^2 \alpha - \sin^2 \alpha = \cos^2 \alpha - \sin^2 \alpha. \text{ Верное тождество.}$$

$$4) (\sin^2 \alpha - \cos^2 \alpha)^2 + 2\sin^2 \alpha \cdot \cos^2 \alpha = \sin^4 \alpha + \cos^4 \alpha;$$

$$\sin^4 \alpha - 2\sin^2 \alpha \cdot \cos^2 \alpha + \cos^4 \alpha + 2\sin^2 \alpha \cdot \cos^2 \alpha = \sin^4 \alpha + \cos^4 \alpha;$$

$$\sin^4 \alpha + \cos^4 \alpha = \sin^4 \alpha + \cos^4 \alpha. \text{ Верное тождество.}$$

$$5) \frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} = \frac{2}{\sin \alpha};$$

$$\frac{\sin^2 \alpha + (1 + \cos \alpha)^2}{(1 + \cos \alpha)\sin \alpha} = \frac{2}{\sin \alpha};$$

$$\frac{\sin^2 \alpha + 1 + 2\cos \alpha + \cos^2 \alpha}{(1 + \cos \alpha)\sin \alpha} = \frac{2}{\sin \alpha};$$

$$\frac{2(1 + \cos \alpha)}{(1 + \cos \alpha)\sin \alpha} = \frac{2}{\sin \alpha}; \quad \frac{2}{\sin \alpha} = \frac{2}{\sin \alpha}. \text{ Верное тождество.}$$

$$6) \frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha};$$

$$\frac{\sin \alpha(1 + \cos \alpha)}{(1 + \cos \alpha)(1 - \cos \alpha)} = \frac{1 + \cos \alpha}{\sin \alpha};$$

$$\frac{\sin \alpha(1 + \cos \alpha)}{1 - \cos^2 \alpha} = \frac{1 + \cos \alpha}{\sin \alpha};$$

$$\frac{\sin \alpha(1 + \cos \alpha)}{\sin^2 \alpha} = \frac{1 + \cos \alpha}{\sin \alpha};$$

$$\frac{1 + \cos \alpha}{\sin \alpha} = \frac{1 + \cos \alpha}{\sin \alpha}. \text{ Верное тождество.}$$

$$7) \frac{1}{1 + \operatorname{tg}^2 \alpha} + \frac{1}{1 + \operatorname{ctg}^2 \alpha} = 1;$$

$$\cos^2 \alpha + \sin^2 \alpha = 1; \quad 1 = 1, \text{ ч.т.д.}$$

$$8) \operatorname{tg}^2 \alpha - \sin^2 \alpha = \operatorname{tg}^2 \alpha \cdot \sin^2 \alpha;$$

$$\frac{\sin^2 \alpha}{\cos^2 \alpha} - \sin^2 \alpha = \operatorname{tg}^2 \alpha \cdot \sin^2 \alpha;$$

$$\frac{\sin^2 \alpha - \sin^2 \alpha \cdot \cos^2 \alpha}{\cos^2 \alpha} = \operatorname{tg}^2 \alpha \cdot \sin^2 \alpha;$$

$$\frac{\sin^2 \alpha(1 - \cos^2 \alpha)}{\cos^2 \alpha} = \operatorname{tg}^2 \alpha \cdot \sin^2 \alpha;$$

$$\operatorname{tg}^2 \alpha \cdot \sin^2 \alpha = \operatorname{tg}^2 \alpha \cdot \sin^2 \alpha, \text{ ч.т.д.}$$

283.

$$1) \frac{(\sin \alpha + \cos \alpha)^2}{\sin^2 \alpha} - (1 + \operatorname{ctg}^2 \alpha) = \frac{1 - 2 \sin \alpha \cos \alpha}{\sin^2 \alpha} - \frac{1}{\sin^2 \alpha} = \\ = \frac{2 \sin \alpha \cos \alpha}{\sin^2 \alpha} = 2 \operatorname{ctg} \alpha ;$$

$$\operatorname{ctg} \frac{\pi}{3} = \frac{1}{\sqrt{3}} ;$$

$$2 \operatorname{ctg} \frac{\pi}{3} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} ;$$

$$2) (1 + \operatorname{tg}^2 \alpha) - \frac{(\sin \alpha - \cos \alpha)^2}{\cos^2 \alpha} = \frac{1}{\cos^2 \alpha} - \frac{1 + 2 \sin \alpha \cos \alpha}{\cos^2 \alpha} = \\ = \frac{2 \sin \alpha \cos \alpha}{\cos^2 \alpha} = 2 \operatorname{tg} \alpha ;$$

$$\operatorname{tg} \frac{\pi}{6} = \frac{1}{\sqrt{3}} ;$$

$$2 \operatorname{tg} \frac{\pi}{6} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} .$$

284.

$$\sin \alpha - \cos \alpha = 0,6 .$$

Возведем в квадрат

$$(\sin \alpha - \cos \alpha)^2 = 0,36 ;$$

$$\sin^2 \alpha - 2 \sin \alpha \cos \alpha + \cos^2 \alpha = 0,36 .$$

Т.к. $\sin^2 \alpha + \cos^2 \alpha = 1$, то

$$1 - 2 \sin \alpha \cos \alpha = 0,36 ;$$

$$2 \sin \alpha \cos \alpha = 1 - 0,36 = 0,64 ;$$

$$\sin \alpha \cos \alpha = 0,32 .$$

285.

$$\cos^3 \alpha - \sin^3 \alpha = (\cos \alpha - \sin \alpha)(\cos^2 \alpha + \cos \alpha \cdot \sin \alpha + \sin^2 \alpha) ;$$

$$\cos^3 \alpha - \sin^3 \alpha = 0,2 \cdot (1 + \cos \alpha \cdot \sin \alpha) ;$$

т.к. $\cos \alpha - \sin \alpha = 0,2$. Возведем в квадрат

$$(\cos \alpha - \sin \alpha)^2 = 0,04 ;$$

$$\cos^2 \alpha - 2 \cos \alpha \sin \alpha + \sin^2 \alpha = 0,04 ;$$

$$1 - 2 \cos \alpha \sin \alpha = 0,04 ;$$

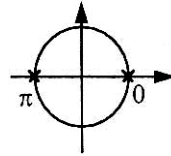
$$\cos \alpha \sin \alpha = 0,48 , \text{ то}$$

$$\cos^3 \alpha - \sin^3 \alpha = 0,2 \cdot (1 + 0,48) = 0,2 \cdot 1,48 = 0,296 .$$

286.

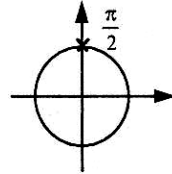
$$\begin{aligned} 1) & 3\cos^2x - 2\sin x = 3 - 3\sin^2x; \\ & 3\cos^2x + 3\sin^2x - 3 - 2\sin x = 0; \\ & 2\sin x = 0; \\ & \sin x = 0. \end{aligned}$$

Тогда $x = \pi n, n \in \mathbb{Z}$.



$$\begin{aligned} 2) & \cos^2x - \sin^2x = 2\sin x - 1 - 2\sin^2x; \\ & \cos^2x - \sin^2x + 1 + 2\sin^2x = 2\sin x; \\ & 2 = 2\sin x; \\ & \sin x = 1. \end{aligned}$$

Значит $x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$.



287.

$$\begin{aligned} 1) & \cos\left(-\frac{\pi}{6}\right)\sin\left(-\frac{\pi}{3}\right) + \operatorname{tg}\left(-\frac{\pi}{4}\right) = -\cos\frac{\pi}{6} \cdot \sin\frac{\pi}{3} - \operatorname{tg}\frac{\pi}{4} = \\ & = -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - 1 = -\frac{3}{4} - 1 = -1\frac{3}{4}; \end{aligned}$$

$$2) \frac{1 + \operatorname{tg}^2(30^\circ)}{1 + \operatorname{ctg}^2(30^\circ)} = \frac{1 + \operatorname{tg}^2 30^\circ}{1 + \operatorname{ctg}^2 30^\circ} = \frac{1 + \frac{1}{3}}{1 + 3} = \frac{3 + 1}{3 \cdot 4} = \frac{4}{3 \cdot 4} = \frac{1}{3};$$

$$\begin{aligned} 3) & 2\sin\left(-\frac{\pi}{6}\right)\cos\left(-\frac{\pi}{6}\right) + \operatorname{tg}\left(-\frac{\pi}{3}\right) + \sin^2\left(-\frac{\pi}{4}\right) = \\ & = -2\sin\frac{\pi}{6}\cos\frac{\pi}{6} - \operatorname{tg}\frac{\pi}{3} + \sin^2\frac{\pi}{4} = -2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \sqrt{3} + \left(\frac{\sqrt{2}}{2}\right)^2 = \\ & = -\frac{\sqrt{3}}{2} - \frac{2\sqrt{3}}{2} + \frac{1}{2} = \frac{1 - 3\sqrt{3}}{2}; \end{aligned}$$

$$\begin{aligned} 4) & \cos(-\pi) + \operatorname{ctg}\left(-\frac{\pi}{2}\right) - \sin\left(\frac{3}{2}\pi\right) + \operatorname{ctg}\left(-\frac{\pi}{4}\right) = \\ & = \cos\pi - \operatorname{ctg}\frac{\pi}{2} + \sin\frac{3}{2}\pi - \operatorname{ctg}\frac{\pi}{4} = -1 + 0 + (-1) - 1 = -3. \end{aligned}$$

288.

$$\begin{aligned} \operatorname{tg}(-\alpha) \cdot \cos\alpha + \sin\alpha &= -\sin\alpha + \sin\alpha = 0; \\ \cos\alpha - \operatorname{ctg}\alpha(-\sin\alpha) &= \cos\alpha + \cos\alpha = 2\cos\alpha; \end{aligned}$$

$$\frac{\cos(-\alpha) + \sin(-\alpha)}{\cos^2 \alpha - \sin^2 \alpha} = \frac{\cos \alpha - \sin \alpha}{(\cos \alpha + \sin \alpha)(\cos \alpha - \sin \alpha)} = \frac{1}{\cos \alpha + \sin \alpha};$$

$$\operatorname{tg}(-\alpha) \cdot \operatorname{ctg}(-\alpha) + \cos^2(-\alpha) + \sin^2 \alpha = 1 + 1 = 2.$$

289.

$$\frac{\cos^2 \alpha - \sin^2 \alpha}{\cos \alpha + \sin(-\alpha)} + \operatorname{tg}(-\alpha) \cos(-\alpha) =$$

$$= \frac{(\cos \alpha + \sin \alpha)(\cos \alpha - \sin \alpha)}{\cos \alpha - \sin \alpha} \sin \alpha =$$

$$= \cos \alpha + \sin \alpha - \sin \alpha = \cos \alpha.$$

290.

$$1) \frac{3 - \sin\left(-\frac{\pi}{3}\right) - \cos^2\left(-\frac{\pi}{3}\right)}{2 \cos\left(-\frac{\pi}{4}\right)} = \frac{3 + \sin \frac{\pi}{3} - \cos^2 \frac{\pi}{3}}{2 \cos \frac{\pi}{4}} = \frac{3 + \frac{\sqrt{3}}{2} - \frac{1}{4}}{2 \cdot \frac{\sqrt{2}}{2}} = \frac{11 + 2\sqrt{3}}{4 \cdot \sqrt{2}};$$

$$2) 2 \sin\left(-\frac{\pi}{6}\right) - 3 \operatorname{ctg}\left(-\frac{\pi}{4}\right) + 7,5 \operatorname{tg}(-\pi) + \frac{1}{8} \cos\left(-\frac{3}{2}\pi\right) =$$

$$= 2 \cdot \left(-\frac{1}{2}\right) - 3 \cdot (-1) + 7,5 \cdot 0 + \frac{1}{8} \cdot 0 = -1 + 3 = 2.$$

291.

$$1) \frac{\sin^3(-\alpha) + \cos^3(-\alpha)}{1 - \sin(-\alpha) \cos(-\alpha)} =$$

$$= \frac{(\cos \alpha - \sin \alpha)(\cos^2 \alpha + \cos \alpha \sin \alpha + \sin^2 \alpha)}{1 + \sin \alpha \cos \alpha} =$$

$$= \frac{(\cos \alpha - \sin \alpha)(1 + \cos \alpha \sin \alpha)}{(1 + \cos \alpha \sin \alpha)} = \cos \alpha - \sin \alpha;$$

$$2) \frac{1 - (\sin \alpha + \cos(-\alpha))^2}{-\sin(-\alpha)} = \frac{1 - (1 + 2 \sin \alpha \cos \alpha)}{\sin \alpha} = \frac{-2 \sin \alpha \cos \alpha}{\sin \alpha} = -2 \cos \alpha.$$

292.

$$1) \sin(-x) = 1;$$

$$\sin x = -1.$$

$$2) \cos(-2x) = 0;$$

$$\cos 2x = 0;$$

$$\text{Тогда } x = -\frac{\pi}{2} + 2\pi n, n \in \wedge.$$

$$2x = \frac{\pi}{2} + \pi n.$$

$$\text{Значит, } x = \frac{\pi}{4} + \frac{\pi n}{2}, n \in \wedge.$$

$$3) \cos(-2x) = 1;$$

$$\cos 2x = 1;$$

$$2x = 2\pi n;$$

$$\text{и } x = \pi n, n \in \mathbb{Z}.$$

$$5) \sin(-x) = \sin \frac{3}{2} \pi;$$

$$-\sin x = -1; \sin x = 1.$$

$$\text{Получим } x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}.$$

$$4) \sin(-2x) = 0;$$

$$2x = 2\pi n.$$

$$\text{Поэтому } x = \frac{\pi n}{2}, n \in \mathbb{Z}.$$

$$6) \cos(-x) = \cos \pi;$$

$$\cos x = -1.$$

$$\text{Тогда } x = \pi + 2\pi n, n \in \mathbb{Z}.$$

293.

$$1) \cos 135^\circ = \cos(90^\circ + 45^\circ) = \cos 90^\circ \cos 45^\circ - \sin 90^\circ \cdot \sin 45^\circ = \\ = 0 \cdot \frac{\sqrt{2}}{2} - 1 \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2};$$

$$2) \cos 120^\circ = \cos(90^\circ + 30^\circ) = \cos 90^\circ \cos 30^\circ - \sin 90^\circ \sin 30^\circ = \\ = 0 \cdot \frac{\sqrt{3}}{2} - 1 \cdot \frac{1}{2} = -\frac{1}{2};$$

$$3) \cos 150^\circ = \cos(90^\circ + 60^\circ) = \cos 90^\circ \cos 60^\circ - \sin 90^\circ \sin 60^\circ = \\ = 0 \cdot \frac{1}{2} - 1 \cdot \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2};$$

$$4) \cos 240^\circ = \cos(180^\circ + 60^\circ) = \cos 180^\circ \cos 60^\circ - \sin 180^\circ \sin 60^\circ = \\ = -1 \cdot \frac{1}{2} - 0 \cdot \frac{\sqrt{3}}{2} = -\frac{1}{2}.$$

294.

$$1) \cos 57^\circ 30' \cdot \cos 27^\circ 30' + \sin 57^\circ 30' \cdot \sin 27^\circ 30' = \\ = \cos(57^\circ 30' - 27^\circ 30') = \cos 30^\circ = \frac{\sqrt{3}}{2};$$

$$2) \cos 19^\circ 30' \cdot \cos 25^\circ 30' - \sin 19^\circ 30' \cdot \sin 25^\circ 30' = \\ = \cos(19^\circ 30' - 25^\circ 30') = \cos 45^\circ = \frac{\sqrt{2}}{2};$$

$$3) \cos \frac{7\pi}{9} \cdot \cos \frac{11\pi}{9} - \sin \frac{7\pi}{9} \cdot \sin \frac{11\pi}{9} = \cos \left(\frac{7\pi}{9} + \frac{11\pi}{9} \right) = \cos 2\pi = 1;$$

$$4) \cos \frac{8\pi}{7} \cdot \cos \frac{\pi}{7} + \sin \frac{8\pi}{7} \cdot \sin \frac{\pi}{7} = \cos \left(\frac{8\pi}{7} + \frac{\pi}{7} \right) = \cos \pi = -1.$$

295.

1) Т.к. $0 < \alpha < \frac{\pi}{2}$, то

$\cos \alpha > 0$, тогда

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{\frac{2}{3}};$$

$$\begin{aligned} \cos\left(\frac{\pi}{3} + \alpha\right) &= \cos \frac{\pi}{3} \cdot \cos \alpha - \sin \frac{\pi}{3} \cdot \sin \alpha = \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{\sqrt{3}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}} = \frac{\sqrt{2} - \sqrt{3}}{2\sqrt{3}}. \end{aligned}$$

2) Т.к. $\frac{\pi}{2} < \alpha < \pi$, то

$\sin \alpha > 0$, тогда

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{\sqrt{3}};$$

$$\cos\left(\alpha - \frac{\pi}{4}\right) = \cos \alpha \cdot \cos \frac{\pi}{4} + \sin \alpha \cdot \sin \frac{\pi}{4} = -\frac{1}{3} \cdot \frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{3} \cdot \frac{\sqrt{2}}{2} = \frac{4 - \sqrt{2}}{6}.$$

296.

1) $\cos 3\alpha \cdot \cos \alpha - \sin \alpha \cdot \sin 3\alpha = \cos(3\alpha + \alpha) = \cos 4\alpha$;

2) $\cos 5\beta \cdot \cos 2\beta + \sin 5\beta \cdot \sin 2\beta = \cos(5\beta - 2\beta) = \cos 3\beta$;

3) $\cos\left(\frac{\pi}{7} + \alpha\right) \cos\left(\frac{5\pi}{14} - \alpha\right) \sin\left(\frac{\pi}{7} + \alpha\right) \sin\left(\frac{5\pi}{14} - \alpha\right) =$
 $= \cos\left(\frac{\pi}{7} + \alpha + \frac{5\pi}{14} - \alpha\right) = \cos \frac{\pi}{2} = 0$;

4) $\cos\left(\frac{7\pi}{5} + \alpha\right) \cdot \cos\left(\frac{2\pi}{5} + \alpha\right) + \sin\left(\frac{7\pi}{5} + \alpha\right) \cdot \sin\left(\frac{2\pi}{5} + \alpha\right) =$
 $= \cos\left(\frac{7\pi}{5} + \alpha - \frac{2\pi}{5} - \alpha\right) = \cos \pi = -1$.

297.

1) $\cos(\alpha + \beta) + \cos\left(\frac{\pi}{2} - \alpha\right) \cos\left(\frac{\pi}{2} - \beta\right) = \cos \alpha \cdot \cos \beta -$
 $-\sin \alpha \cdot \sin \beta + \sin \alpha \cdot \sin \beta = \cos \alpha \cdot \cos \beta$;

$$\begin{aligned}
 & 2) \sin\left(\frac{\pi}{2} - \alpha\right) \sin\left(\frac{\pi}{2} - \beta\right) - \cos(\alpha - \beta) = \left(\sin\frac{\pi}{2} \cdot \cos\alpha - \cos\frac{\pi}{2} \cdot \sin\alpha\right) \times \\
 & \times \left(\sin\frac{\pi}{2} \cdot \cos\beta - \cos\frac{\pi}{2} \cdot \sin\beta\right) - (\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta) = \\
 & = \cos\alpha \cdot \cos\beta - \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta = -\sin\alpha \cdot \sin\beta.
 \end{aligned}$$

298.

$$1) \sin 73^\circ \cdot \cos 17^\circ + \cos 73^\circ \cdot \sin 17^\circ = \sin(73^\circ + 17^\circ) = \sin 90^\circ = 1;$$

$$2) \sin 73^\circ \cdot \cos 13^\circ - \cos 73^\circ \cdot \sin 13^\circ = \sin(73^\circ - 13^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2};$$

$$3) \sin \frac{5\pi}{12} \cdot \cos \frac{\pi}{12} + \sin \frac{\pi}{12} \cdot \cos \frac{5\pi}{12} = \sin\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) = \sin \frac{\pi}{2} = 1;$$

$$4) \sin \frac{7\pi}{12} \cdot \cos \frac{\pi}{12} - \sin \frac{\pi}{12} \cdot \cos \frac{7\pi}{12} = \sin\left(\frac{7\pi}{12} - \frac{\pi}{12}\right) = \sin \frac{\pi}{2} = 1.$$

299.

$$1) \text{Т.к. } \pi < \alpha < \frac{3\pi}{2}, \text{ то}$$

$\sin \alpha < 0$, тогда

$$\sin \alpha = -\sqrt{1 - \cos^2 \alpha} = -\sqrt{1 - \frac{9}{25}} = -\frac{4}{5};$$

$$\begin{aligned}
 \sin\left(\alpha + \frac{\pi}{6}\right) &= \sin \alpha \cdot \cos \frac{\pi}{6} + \cos \alpha \cdot \sin \frac{\pi}{6} = -\frac{4}{5} \cdot \frac{\sqrt{3}}{2} - \frac{3}{5} \cdot \frac{1}{2} = \\
 &= \frac{-4\sqrt{3} - 3}{10} = -\frac{4\sqrt{3} + 3}{10}.
 \end{aligned}$$

$$2) \text{Т.к. } \frac{\pi}{2} < \alpha < \pi, \text{ то}$$

$\cos \alpha < 0$, тогда

$$\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \frac{2}{9}} = \frac{\sqrt{7}}{3};$$

$$\begin{aligned}
 \sin\left(\frac{\pi}{4} - \alpha\right) &= \sin \frac{\pi}{4} \cdot \cos \alpha - \cos \frac{\pi}{4} \cdot \sin \alpha = \frac{\sqrt{2}}{2} \cdot \left(\frac{\sqrt{7}}{3}\right) - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{3} = \\
 &= \frac{-\sqrt{14} - 2}{6} = -\frac{\sqrt{14} + 2}{6}.
 \end{aligned}$$

300.

$$1) \sin(\alpha + \beta) + \sin(-\alpha)\cos(-\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta - \sin\alpha \cdot \cos\beta = \cos\alpha \cdot \sin\beta;$$

$$2) \cos(-\alpha)\sin(-\beta) - \sin(\alpha - \beta) = \\ = -\cos\alpha \cdot \sin\beta - (\sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta) = -\cos\alpha \cdot \sin\beta - \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta = -\sin\alpha \cdot \cos\beta;$$

3)

$$3) \cos\left(\frac{\pi}{2} - \alpha\right)\sin\left(\frac{\pi}{2} - \beta\right) - \sin(\alpha - \beta) = \left(\cos\frac{\pi}{2}\cos\alpha + \sin\frac{\pi}{2}\sin\alpha\right) \times \\ \times \left(\sin\frac{\pi}{2}\cos\beta - \cos\frac{\pi}{2}\sin\beta\right) - \sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin\alpha\cos\beta - \\ - \sin\alpha\cos\beta + \cos\alpha\sin\beta = \cos\alpha\sin\beta;$$

$$4) \sin(\alpha + \beta) + \sin\left(\frac{\pi}{2} - \alpha\right)\sin(-\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta - \\ - \cos\alpha\sin\beta = \sin\alpha\cos\beta.$$

301.

Т.к. $\frac{3\pi}{2} < \alpha < 2\pi$, то

$$\cos\alpha > 0,$$

тогда

$$\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}.$$

Т.к. $0 < \beta < \frac{\pi}{2}$, то

$$\cos\beta > 0,$$

тогда

$$\cos\beta = \sqrt{1 - \sin^2\beta} = \sqrt{1 - \frac{64}{289}} = \frac{15}{17};$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta = \\ = \frac{4}{5} \cdot \frac{15}{17} - \left(-\frac{3}{5}\right) \cdot \frac{8}{17} = \frac{60}{85} + \frac{24}{85} = \frac{84}{85};$$

$$\cos(\alpha - \beta) = \frac{4}{5} \cdot \frac{15}{17} + \left(-\frac{3}{5}\right) \cdot \frac{8}{17} = \frac{60}{85} - \frac{24}{85} = \frac{36}{85}.$$

302.

Т.к. $\frac{\pi}{2} < \alpha < \pi$, то $\sin \alpha > 0$;

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - 0,64} = \sqrt{0,36} = 0,6.$$

Т.к. $\pi < \beta < \frac{3\pi}{2}$,

то $\cos \beta < 0$;

$$\cos \beta = -\sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \frac{144}{169}} = -\frac{5}{13};$$

$$\begin{aligned} \sin(\alpha - \beta) &= \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta = \\ &= 0,6 \cdot \left(-\frac{5}{13}\right) - (-0,8) \cdot \left(-\frac{12}{13}\right) = \frac{-15}{65} - \frac{48}{65} = -\frac{63}{65}. \end{aligned}$$

303.

$$\begin{aligned} 1) \cos\left(\frac{2}{3}\pi - \alpha\right) + \cos\left(\alpha + \frac{\pi}{3}\right) &= \cos \frac{2\pi}{3} \cdot \cos \alpha + \sin \frac{2\pi}{3} \cdot \sin \alpha + \\ + \cos \frac{\pi}{3} \cdot \cos \alpha - \sin \frac{\pi}{3} \cdot \sin \alpha &= -\frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha = 0; \end{aligned}$$

$$\begin{aligned} 2) \sin\left(\alpha + \frac{2}{3}\pi\right) - \sin\left(\frac{\pi}{3} - \alpha\right) &= \sin \alpha \cdot \cos \frac{2\pi}{3} + \cos \alpha \cdot \sin \frac{2\pi}{3} - \\ - \sin \frac{\pi}{3} \cdot \cos \alpha + \cos \frac{\pi}{3} \cdot \sin \alpha &= -\frac{1}{2} \sin \alpha + \frac{\sqrt{3}}{2} \cos \alpha - \frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha = 0; \end{aligned}$$

$$\begin{aligned} 3) \frac{2 \cos \alpha \sin \beta + \sin(\alpha - \beta)}{2 \cos \alpha \cos \beta - \cos(\alpha - \beta)} &= \frac{2 \cos \alpha \sin \beta + \sin \alpha \cos \beta - \cos \alpha \sin \beta}{2 \cos \alpha \cos \beta - \cos \alpha \cos \beta - \sin \alpha \sin \beta} = \\ = \frac{\cos \alpha \sin \beta + \sin \alpha \cos \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \operatorname{tg}(\alpha + \beta); \end{aligned}$$

$$\begin{aligned} 4) \frac{\cos \alpha \cos \beta - \cos(\alpha + \beta)}{\cos(\alpha - \beta) - \sin \alpha \sin \beta} &= \frac{\cos \alpha \cos \beta - \cos \alpha \cos \beta + \sin \alpha \sin \beta}{\cos \alpha \cos \beta + \sin \alpha \sin \beta - \sin \alpha \sin \beta} = \\ = \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta} &= \operatorname{tg} \alpha \cdot \operatorname{tg} \beta. \end{aligned}$$

304.

$$\begin{aligned} 1) \sin(\alpha - \beta) \cdot \cos(\alpha + \beta) &= (\sin \alpha \cos \beta - \cos \alpha \sin \beta)(\sin \alpha \cos \beta + \\ + \cos \alpha \sin \beta) &= \sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta = \sin^2 \alpha (1 - \sin^2 \beta) - (1 - \\ - \sin^2 \alpha) \sin^2 \beta &= \sin^2 \alpha - \sin^2 \alpha \cdot \sin^2 \beta - \sin^2 \beta + \sin^2 \alpha \cdot \sin^2 \beta = \sin^2 \alpha - \sin^2 \beta; \end{aligned}$$

$$\begin{aligned}
 & 2) \sin(\alpha - \beta) \cdot \cos(\alpha + \beta) = (\cos\alpha \cos\beta - \sin\alpha \sin\beta)(\cos\alpha \cos\beta + \\
 & + \sin\alpha \sin\beta) = \cos^2\alpha \cos^2\beta - \sin^2\alpha \sin^2\beta = \cos^2\alpha(1 - \sin^2\beta) - \\
 & - (1 - \cos^2\alpha) \sin^2\beta = \cos^2\alpha - \cos^2\alpha \cdot \sin^2\beta - \sin^2\beta + \cos^2\alpha \cdot \sin^2\beta = \\
 & = \cos^2\alpha - \sin^2\beta;
 \end{aligned}$$

$$\begin{aligned}
 3) \frac{\sqrt{2} \cos \alpha - 2 \cos\left(\frac{\pi}{4} - \alpha\right)}{2 \sin\left(\frac{\pi}{6} + \alpha\right) - \sqrt{3} \sin \alpha} &= \frac{\sqrt{2} \cos \alpha - 2\left(\frac{\sqrt{2}}{2} \cos \alpha + \frac{\sqrt{2}}{2} \sin \alpha\right)}{2\left(\frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha\right) - \sqrt{3} \sin \alpha} = \\
 &= \frac{\sqrt{2} \cos \alpha - \sqrt{2} \cos \alpha - \sqrt{2} \sin \alpha}{\cos \alpha + \sqrt{3} \sin \alpha - \sqrt{3} \sin \alpha} = \frac{-\sqrt{2} \sin \alpha}{\cos \alpha} = -\sqrt{2} \operatorname{tg} \alpha;
 \end{aligned}$$

$$\begin{aligned}
 4) \frac{\cos \alpha - 2 \cos\left(\frac{\pi}{3} + \alpha\right)}{2 \sin\left(\alpha - \frac{\pi}{6}\right) - \sqrt{3} \sin \alpha} &= \frac{\cos \alpha - 2\left(\frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha\right)}{2\left(\frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \cos \alpha\right) - \sqrt{3} \sin \alpha} = \\
 &= \frac{\cos \alpha - \cos \alpha + \sqrt{3} \sin \alpha}{\sqrt{3} \sin \alpha - \cos \alpha - \sqrt{3} \sin \alpha} = \frac{\sqrt{3} \sin \alpha}{-\cos \alpha} = -\sqrt{3} \operatorname{tg} \alpha.
 \end{aligned}$$

305.

$$\begin{aligned}
 1) \cos 6x \cdot \cos 5x + \sin 6x \cdot \sin 5x &= -1; \\
 \cos(6x - 5x) &= -1. \text{ Тогда } \cos x = -1;
 \end{aligned}$$

$$x = \pi + 2\pi n, n \in \mathbb{Z};$$

$$\begin{aligned}
 2) \sin 3x \cdot \cos 5x - \sin 5x \cdot \cos 3x &= -1; \\
 \sin(3x - 5x) &= -1; \quad -\sin 2x = -1;
 \end{aligned}$$

$$\sin 2x = 1. \text{ Значит, } 2x = \frac{\pi}{2} + 2\pi n;$$

$$x = \frac{\pi}{4} + 2\pi n, n \in \mathbb{Z};$$

$$3) \sqrt{2} \cos\left(\frac{\pi}{4} + x\right) - \cos x = 1;$$

$$\sqrt{2} \left(\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x \right) - \cos x = 1;$$

$$\cos x - \sin x - \cos x = 1;$$

$$\sin x = -1. \text{ Поэтому } x = -\frac{\pi}{2} + 2\pi n, n \in \mathbb{Z};$$

$$4) \sqrt{2} \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sin \frac{x}{2} = 1; \quad \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos \frac{x}{2} - \frac{\sqrt{2}}{2} \sin \frac{x}{2} \right) + \sin \frac{x}{2} = 1;$$

$$\cos \frac{x}{2} - \sin \frac{x}{2} + \sin \frac{x}{2} = 1;$$

$$\cos \frac{x}{2} = 1.$$

Значит, $\frac{x}{2} = 2\pi n$ и $x = 4\pi n$, $n \in \mathbb{Z}$.

306.

$$1) \frac{\operatorname{tg} 29^\circ + \operatorname{tg} 31^\circ}{1 - \operatorname{tg} 29^\circ \cdot \operatorname{tg} 31^\circ} = \operatorname{tg}(29^\circ + 31^\circ) = \operatorname{tg} 60^\circ = \sqrt{3};$$

$$2) \frac{\operatorname{tg} \frac{7\pi}{16} - \operatorname{tg} \frac{3\pi}{16}}{1 + \operatorname{tg} \frac{7\pi}{16} \cdot \operatorname{tg} \frac{3\pi}{16}} = \operatorname{tg} \left(\frac{7\pi}{16} - \frac{3\pi}{16} \right) = \operatorname{tg} \frac{\pi}{4} = 1.$$

307.

$$1) \frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{\sin\alpha \cos\beta + \cos\alpha \sin\beta}{\sin\alpha \cos\beta - \cos\alpha \sin\beta} = \frac{\frac{\sin\alpha \cos\beta}{\cos\alpha \cos\beta} + \frac{\cos\alpha \sin\beta}{\cos\alpha \cos\beta}}{\frac{\sin\alpha \cos\beta}{\cos\alpha \cos\beta} - \frac{\cos\alpha \sin\beta}{\cos\alpha \cos\beta}} = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{\operatorname{tg}\alpha - \operatorname{tg}\beta};$$

$$2) \frac{\cos(\alpha - \beta)}{\cos(\alpha + \beta)} = \frac{\cos\alpha \cos\beta + \sin\alpha \sin\beta}{\cos\alpha \cos\beta - \sin\alpha \sin\beta} = \frac{\frac{\cos\alpha \cos\beta}{\sin\alpha \sin\beta} + \frac{\sin\alpha \sin\beta}{\sin\alpha \sin\beta}}{\frac{\cos\alpha \cos\beta}{\sin\alpha \sin\beta} - \frac{\sin\alpha \sin\beta}{\sin\alpha \sin\beta}} = \frac{\operatorname{ctg}\alpha \cdot \operatorname{ctg}\beta + 1}{\operatorname{ctg}\alpha \cdot \operatorname{ctg}\beta - 1}.$$

308.

$$1) 2\sin 15^\circ \cos 15^\circ = \sin 2 \cdot 15^\circ = \sin 30^\circ = \frac{1}{2};$$

$$2) \cos^2 15^\circ - \sin^2 15^\circ = \cos 2 \cdot 15^\circ - \cos 30^\circ = \frac{\sqrt{3}}{2};$$

$$3) (\cos 75^\circ - \sin 75^\circ)^2 = \cos^2 75^\circ - 2\sin 75^\circ \cos 75^\circ + \sin^2 75^\circ = 1 - \sin 150^\circ = 1 - \sin 30^\circ = 1 - \frac{1}{2} = \frac{1}{2};$$

$$4) (\cos 15^\circ + \sin 15^\circ)^2 = \cos^2 15^\circ + 2\sin 15^\circ \cos 15^\circ + \sin^2 15^\circ = 1 + \sin 30^\circ = 1 + \frac{1}{2} = \frac{3}{2}.$$

309.

$$1) 2 \sin \frac{\pi}{8} \cos \frac{\pi}{8} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}; \quad 2) \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2};$$

$$3) \sin \frac{\pi}{8} \cos \frac{\pi}{8} + \frac{1}{4} = \frac{1}{2} \sin \frac{\pi}{4} + \frac{1}{4} = \frac{\sqrt{2}}{4} + \frac{1}{4} = \frac{\sqrt{2}+1}{4};$$

$$4) \frac{\sqrt{2}}{2} - \left(\cos \frac{\pi}{8} + \sin \frac{\pi}{8} \right)^2 = \frac{\sqrt{2}}{2} - \left(1 + 2 \sin \frac{\pi}{8} \cos \frac{\pi}{8} \right) = \\ = \frac{\sqrt{2}}{2} - \left(1 + \sin \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2} - \left(1 + \frac{\sqrt{2}}{2} \right) = \frac{\sqrt{2}}{2} - 1 - \frac{\sqrt{2}}{2} = -1.$$

310.

1) Т.к. $\frac{\pi}{2} < \alpha < \pi$, то $\cos \alpha < 0$, тогда

$$\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \frac{9}{25}} = -\frac{4}{5};$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = 2 \cdot \frac{3}{5} \left(-\frac{4}{5} \right) = -\frac{24}{25}.$$

2) Т.к. $\pi < \alpha < \frac{3\pi}{2}$, то

$$\sin \alpha < 0, \text{ тогда } \sin \alpha = -\sqrt{1 - \frac{16}{25}} = -\frac{3}{5};$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = 2 \cdot \left(-\frac{3}{5} \right) \cdot \left(-\frac{4}{5} \right) = \frac{24}{25}.$$

311.

$$1) \sin^2 \alpha = 1 - \cos^2 \alpha;$$

$$2) \cos^2 \alpha = 1 - \sin^2 \alpha;$$

$$\sin^2 \alpha = 1 - \frac{16}{25} = \frac{9}{25}.$$

$$\cos^2 \alpha = 1 - \frac{9}{25} = \frac{16}{25}.$$

Т.к. $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$, то

Т.к. $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$, то

$$\cos 2\alpha = \frac{16}{25} - \frac{9}{25} = \frac{7}{25};$$

$$\cos 2\alpha = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}.$$

312.

$$1) \sin \alpha \cos \alpha = \frac{2 \sin \alpha \cos \alpha}{2} = \frac{\sin 2\alpha}{2};$$

$$2) \cos \alpha \cos \left(\frac{\pi}{2} - \alpha \right) = \sin \alpha \cos \alpha = \frac{\sin 2\alpha}{2};$$

$$3) \cos 4\alpha + \sin^2 2\alpha = \cos^2 2\alpha - \sin^2 2\alpha + \sin^2 2\alpha = \cos^2 2\alpha;$$

$$4) \sin 2\alpha + (\sin\alpha - \cos\alpha)^2 = 2\sin\alpha\cos\alpha + \sin^2\alpha - 2\sin\alpha\cos\alpha + \cos^2\alpha = 1.$$

313.

$$1) \frac{\cos 2\alpha + 1}{2\cos\alpha} = \frac{\cos^2\alpha - \sin^2\alpha + \cos^2\alpha + \sin^2\alpha}{2\cos\alpha} = \frac{2\cos^2\alpha}{2\cos\alpha} = \cos\alpha;$$

$$2) \frac{\sin 2\alpha}{1 - \cos^2\alpha} = \frac{2\sin\alpha\cos\alpha}{\sin^2\alpha} = \frac{2\cos\alpha}{\sin\alpha} = 2\operatorname{ctg}\alpha;$$

$$3) \frac{\sin^2\alpha}{(\sin\alpha + \cos\alpha)^2 - 1} = \frac{\sin^2\alpha}{\sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha - 1} =$$

$$= \frac{\sin^2\alpha}{2\sin\alpha\cos\alpha} = \frac{1}{2}\operatorname{tg}\alpha;$$

$$4) \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha} = \frac{\cos^2\alpha + \sin^2\alpha + \cos^2\alpha - \sin^2\alpha}{\cos^2\alpha + \sin^2\alpha - \cos^2\alpha + \sin^2\alpha} = \frac{2\cos^2\alpha}{2\sin^2\alpha} = \operatorname{ctg}^2\alpha.$$

314.

$$1) (\sin\alpha + \cos\alpha)^2 - 1 = 1 + 2\sin\alpha\cos\alpha - 1 = 2\sin\alpha\cos\alpha = \sin 2\alpha;$$

$$2) (\sin\alpha - \cos\alpha)^2 = \sin^2\alpha - 2\sin\alpha\cos\alpha + \cos^2\alpha = 1 - \sin 2\alpha;$$

$$3) \cos^4\alpha - \sin^4\alpha = (\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha) = \cos 2\alpha;$$

$$4) 2\cos^2\alpha - \cos 2\alpha = 2\cos^2\alpha - \cos^2\alpha + \sin^2\alpha = \cos^2\alpha + \sin^2\alpha = 1.$$

315.

$$1) \sin\alpha + \cos\alpha = \frac{1}{2}.$$

Возведем в квадрат.

$$\text{Получим: } (\sin\alpha + \cos\alpha)^2 = \frac{1}{4};$$

$$1 + 2\sin\alpha\cos\alpha = \frac{1}{4}; \sin 2\alpha = \frac{1}{4} - 1 = -\frac{3}{4}.$$

$$2) \sin\alpha - \cos\alpha = -\frac{1}{3}.$$

Возведем в квадрат

$$(\sin\alpha - \cos\alpha)^2 = \frac{1}{9}; 1 - 2\sin\alpha\cos\alpha = \frac{1}{9}; \sin 2\alpha = 1 - \frac{1}{9} = \frac{8}{9}.$$

316.

$$1) 1 + \cos 2\alpha = \sin^2\alpha + \cos^2\alpha + \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha;$$

$$2) 2\sin^2\alpha = \sin^2\alpha + \cos^2\alpha - \cos^2\alpha + \sin^2\alpha = 1 - \cos 2\alpha.$$

317.

$$\begin{aligned} 1) 2 \cos^2 15^\circ - 1 &= 2 \cos^2 15^\circ - (\sin^2 15^\circ + \cos^2 15^\circ) = \\ &= \cos^2 15^\circ - \sin^2 15^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}; \end{aligned}$$

$$\begin{aligned} 2) 1 - \sin^2 22,5^\circ &= \sin^2 22,5^\circ + \cos^2 22,5^\circ - 2 \sin^2 22,5^\circ = \\ &= \cos^2 22,5^\circ - \sin^2 22,5^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}; \end{aligned}$$

$$\begin{aligned} 3) 2 \cos^2 \frac{\pi}{8} - 1 &= 2 \cos^2 \frac{\pi}{8} - \left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right) = \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} = \\ &= \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}; \end{aligned}$$

$$\begin{aligned} 4) 1 - 2 \sin^2 \frac{\pi}{12} &= \cos^2 \frac{\pi}{12} + \sin^2 \frac{\pi}{12} - 2 \sin^2 \frac{\pi}{12} = \cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12} = \\ &= \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}. \end{aligned}$$

318.

$$\begin{aligned} 1) 1 - 2 \sin^2 5\alpha &= \sin^2 5\alpha + \cos^2 5\alpha - 2 \sin^2 5\alpha = \cos^2 5\alpha - \sin^2 5\alpha = \cos 10\alpha; \\ 2) 2 \cos^2 3\alpha - 1 &= 2 \cos^2 3\alpha - (\sin^2 3\alpha + \cos^2 3\alpha) = \\ &= \cos^2 3\alpha - \sin^2 3\alpha = \cos 6\alpha; \end{aligned}$$

$$3) \frac{1 - \cos 2\alpha}{\sin \frac{\alpha}{2} \cos \frac{\alpha}{2}} = \frac{\sin^2 \alpha + \cos^2 \alpha - \cos^2 \alpha + \sin^2 \alpha}{\frac{1}{2} \sin \alpha} = \frac{4 \sin^2 \alpha}{\sin \alpha} = 4 \sin \alpha;$$

$$4) \frac{2 \cos^2 \frac{\alpha}{2} - 1}{\sin 2\alpha} = \frac{2 \cos^2 \frac{\alpha}{2} - \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}}{2 \sin \alpha \cdot \cos \alpha} = \frac{\cos \alpha}{2 \sin \alpha \cos \alpha} = \frac{1}{2 \sin \alpha}.$$

319.

$$\begin{aligned} 1) \frac{\cos 2\alpha}{\sin \alpha \cos \alpha + \sin^2 \alpha} &= \frac{(\cos \alpha - \sin \alpha)(\cos \alpha + \sin \alpha)}{\sin \alpha (\cos \alpha + \sin \alpha)} = \frac{\cos \alpha - \sin \alpha}{\sin \alpha} = \\ &= \frac{\cos \alpha}{\sin \alpha} - \frac{\sin \alpha}{\sin \alpha} = \operatorname{ctg} \alpha - 1; \end{aligned}$$

$$2) \frac{\sin 2\alpha - 2 \cos \alpha}{\sin \alpha - \sin^2 \alpha} = \frac{2 \cos \alpha (\sin \alpha - 1)}{\sin \alpha (1 - \sin \alpha)} = -\frac{2 \cos \alpha}{\sin \alpha} = -2 \operatorname{ctg} \alpha;$$

$$\begin{aligned} 3) \operatorname{tg} \alpha \cdot (1 + \cos 2\alpha) &= \operatorname{tg} \alpha \cdot (\cos^2 \alpha + \sin^2 \alpha + \cos^2 \alpha - \sin^2 \alpha) = \\ &= \frac{\sin \alpha}{\cos \alpha} \cdot 2 \cos^2 \alpha = 2 \sin \alpha \cos \alpha = \sin 2\alpha; \end{aligned}$$

$$\begin{aligned}
 & 4) \frac{1 - \cos 2\alpha + \sin 2\alpha}{1 + \cos 2\alpha + \sin 2\alpha} = \\
 & = \frac{\cos^2 \alpha + \sin^2 \alpha - \cos^2 \alpha + \sin^2 \alpha + 2 \sin \alpha \cos \alpha}{\cos^2 \alpha + \sin^2 \alpha + \cos^2 \alpha - \sin^2 \alpha + 2 \sin \alpha \cos \alpha} \cdot \frac{\cos \alpha}{\sin \alpha} = \\
 & = \frac{2 \sin \alpha (\sin \alpha + \cos \alpha) \cdot \cos \alpha}{2 \cos \alpha (\sin \alpha + \cos \alpha) \cdot \sin \alpha} = 1.
 \end{aligned}$$

320.

$$1) \sin 2x - 2 \cos x = 0;$$

$$2 \cos x \cdot \sin x - 2 \cos x = 0;$$

$$2 \cos x (\sin x - 1) = 0.$$

$$\text{Тогда } \begin{cases} \cos x = 0 \\ \sin x - 1 = 0 \end{cases}; \begin{cases} x = \frac{\pi}{2} + \pi n; \\ \sin x = 1 \end{cases}; \begin{cases} x = \frac{\pi}{2} + \pi n, & n \in Z \\ x = \frac{\pi}{2} + 2\pi n, & n \in Z \end{cases}.$$

$$\text{Ответ: } \frac{\pi}{2} + \pi n.$$

$$2) \cos 2x + 3 \sin x = 1;$$

$$\cos^2 x - \sin^2 x + 3 \sin x - \sin^2 x - \cos^2 x = 0;$$

$$3 \sin x - 2 \sin^2 x = 0;$$

$$\sin x (-2 \sin x + 3) = 0;$$

$$\begin{cases} \sin x = 0 \\ -2 \sin x + 3 = 0 \end{cases}; \begin{cases} x = \pi n, & n \in Z \\ \sin x = 1,5 \end{cases} \quad \text{— нет решения}$$

$$\text{Ответ: } \pi n; n \in \mathbb{Z}.$$

$$3) 2 \sin x = \sin 2x;$$

$$2 \sin x - 2 \sin x \cdot \cos x = 0;$$

$$2 \sin x (1 - \cos x) = 0;$$

$$\begin{cases} \sin x = 0 \\ 1 - \cos x = 0 \end{cases}; \begin{cases} x = \pi n, & n \in Z \\ \cos x = 1 \end{cases}; \begin{cases} x = \pi n, & n \in Z \\ x = 2\pi n \end{cases}.$$

$$\text{Ответ: } \pi n.$$

$$4) \sin^2 x = -\cos 2x;$$

$$\sin^2 x + \cos^2 x - \sin^2 x = 0;$$

$$\cos^2 x = 0;$$

$$\cos x = 0.$$

$$\text{Ответ: } \frac{\pi}{2} + \pi n; n \in \mathbb{Z}.$$

321.

$$\text{T.k. } \operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}, \text{ то } \operatorname{tg} 2\alpha = \frac{2 \cdot 0,6}{1-0,36} = \frac{1,2}{0,64} = \frac{120}{64} = 1\frac{7}{8}.$$

322.

$$1) \frac{2\operatorname{tg}\frac{\pi}{8}}{1-\operatorname{tg}^2\frac{\pi}{8}} = \operatorname{tg}\frac{\pi}{4} = 1; \quad 2) \frac{6\operatorname{tg}15^\circ}{1-\operatorname{tg}^215^\circ} = 3 \cdot \operatorname{tg}30^\circ = 3 \cdot \frac{1}{\sqrt{3}} = \sqrt{3}.$$

323.

$$1) \sin\frac{13}{2}\pi = \sin\left(6\pi + \frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1;$$

$$2) \sin 17\pi = \sin(18\pi - \pi) = -\sin\pi = 0;$$

$$3) \cos 7\pi = \cos(8\pi - \pi) = \cos\pi = -1;$$

$$4) \cos\frac{11}{2}\pi = \cos\left(6\pi - \frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0;$$

$$5) \sin 720^\circ = \sin(2 \cdot 360^\circ) = 0;$$

$$6) \cos 540^\circ = \cos(360^\circ + 180^\circ) = \cos 180^\circ = -1.$$

324.

$$1) \cos 420^\circ = \cos(360^\circ + 60^\circ) = \cos 60^\circ = \frac{1}{2};$$

$$2) \operatorname{tg} 570^\circ = \operatorname{tg}(3 \cdot 180^\circ + 30^\circ) = \operatorname{tg} 30^\circ = \frac{1}{\sqrt{3}};$$

$$3) \sin 3630^\circ = \sin(10 \cdot 360^\circ + 30^\circ) = \sin 30^\circ = \frac{1}{2};$$

$$4) \operatorname{ctg} 960^\circ = \operatorname{ctg}(5 \cdot 180^\circ + 60^\circ) = \operatorname{ctg} 60^\circ = \frac{1}{\sqrt{3}};$$

$$5) \sin\frac{13\pi}{6} = \sin\left(2\pi + \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2};$$

$$6) \operatorname{tg}\frac{11\pi}{6} = \operatorname{tg}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{tg}\frac{\pi}{6} = -\frac{1}{\sqrt{3}}.$$

325.

$$1) \cos 150^\circ = \cos(90^\circ + 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2};$$

$$2) \sin 135^\circ = \sin(90^\circ + 45^\circ) = \cos 45^\circ = \frac{\sqrt{2}}{2};$$

$$3) \cos 120^\circ = -\cos 60^\circ = -\frac{1}{2};$$

$$4) \sin 315^\circ = \sin (360^\circ - 45^\circ) = -\sin 45^\circ = -\frac{\sqrt{2}}{2}.$$

326.

$$1) \operatorname{tg} \frac{5\pi}{4} = \operatorname{tg} \left(\pi + \frac{\pi}{4} \right) = \operatorname{tg} \frac{\pi}{4} = 1;$$

$$2) \sin \frac{7\pi}{6} = \sin \left(\pi + \frac{\pi}{6} \right) = -\sin \frac{\pi}{6} = -\frac{1}{2};$$

$$3) \cos \frac{5\pi}{3} = \cos \left(2\pi - \frac{\pi}{3} \right) = \cos \frac{\pi}{3} = \frac{1}{2};$$

$$4) \sin \left(-\frac{11\pi}{6} \right) = -\sin \left(2\pi - \frac{\pi}{6} \right) = \sin \frac{\pi}{6} = \frac{1}{2};$$

$$5) \cos \left(-\frac{7\pi}{3} \right) = \cos \left(2\pi + \frac{\pi}{3} \right) = \cos \frac{\pi}{3} = \frac{1}{2};$$

$$6) \operatorname{tg} \left(-\frac{2\pi}{3} \right) = -\operatorname{tg} \left(\pi - \frac{\pi}{3} \right) = \operatorname{tg} \frac{\pi}{3} = \sqrt{3}.$$

327.

$$\begin{aligned} 1) \cos 630^\circ - \sin 1470^\circ - \operatorname{ctg} 1125^\circ &= \cos(720^\circ - 90^\circ) - \\ - \sin(1440^\circ + 30^\circ) - \operatorname{ctg}(1080^\circ + 45^\circ) &= \cos 90^\circ - \sin 30^\circ - \operatorname{ctg} 45^\circ = \\ = 0 - \frac{1}{2} - 1 &= -\frac{3}{2}; \end{aligned}$$

$$\begin{aligned} 2) \operatorname{tg} 1800^\circ - \sin 495^\circ + \cos 945^\circ &= 0 - \sin 135^\circ + \cos 225^\circ = \\ = -\sin(90^\circ + 45^\circ) + \cos(180^\circ + 45^\circ) &= -\cos 45^\circ - \cos 45^\circ = -2 \cdot \frac{\sqrt{2}}{2} = -\sqrt{2}; \end{aligned}$$

$$\begin{aligned} 3) \sin(-7\pi) - 2 \cos \frac{31\pi}{3} - \operatorname{tg} \frac{7\pi}{4} &= -\sin(6\pi + \pi) - \\ - 2 \cos \left(10\pi + \frac{\pi}{3} \right) - \operatorname{tg} \left(2\pi - \frac{\pi}{4} \right) &= -\sin \pi - 2 \cos \frac{\pi}{3} + \operatorname{tg} \frac{\pi}{4} = \\ = 0 - 2 \cdot \frac{1}{2} + 1 &= -1 + 1 = 0; \end{aligned}$$

$$\begin{aligned} 4) \cos(-9\pi) + 2 \sin \left(-\frac{49\pi}{6} \right) - \operatorname{ctg} \left(-\frac{21\pi}{4} \right) &= \cos \pi - 2 \sin \left(8\pi + \frac{\pi}{6} \right) + \\ + \operatorname{ctg} \left(5\pi + \frac{\pi}{4} \right) &= -1 - \sin \frac{\pi}{6} + \operatorname{ctg} \frac{\pi}{4} = -1 - 2 \cdot \frac{1}{2} + 1 = -1 - 1 + 1 = -1. \end{aligned}$$

328.

$$\begin{aligned} 1) \cos^2(\pi - \alpha) + \sin^2(\alpha - \pi) &= \cos^2\alpha + \sin^2\alpha = 1; \\ 2) \cos(\pi - \alpha)\cos(3\pi - \alpha) - \sin(\alpha - \pi)\sin(\alpha - 3\pi) &= \\ = \cos(\pi - \alpha)\cos(3\pi - \alpha) - \sin(\pi - \alpha)\sin(3\pi - \alpha) &= \cos(\pi - \alpha + 3\pi - \alpha) = \\ = \cos(4\pi - 2\alpha) = \cos 2\alpha. \end{aligned}$$

329.

$$\begin{aligned} 1) \cos 723^\circ + \sin 900^\circ &= \cos(360^\circ \cdot 20 + 30^\circ) + \sin(360^\circ \cdot 2 + 180^\circ) = \\ = \cos 30^\circ + \sin 180^\circ &= \frac{\sqrt{3}}{2} + 0 = \frac{\sqrt{3}}{2}; \\ 2) \sin 300^\circ + \operatorname{tg} 150^\circ &= \sin(360^\circ - 60^\circ) + \operatorname{tg}(180^\circ - 30^\circ) = \\ = -\sin 60^\circ - \operatorname{tg} 30^\circ &= -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{3} = \frac{-5\sqrt{3}}{6}; \\ 3) 2 \sin 6,5\pi - \sqrt{3} \sin \frac{19\pi}{3} &= 2 \sin\left(6\pi + \frac{\pi}{2}\right) - \sqrt{3} \sin\left(6\pi + \frac{\pi}{3}\right) = \\ = 2 \sin \frac{\pi}{2} - \sqrt{3} \sin \frac{\pi}{3} &= 2 - \sqrt{3} \cdot \frac{\sqrt{3}}{2} = 2 - \frac{3}{2} = \frac{1}{2}; \\ 4) \sqrt{2} \cos 4,25\pi - \frac{1}{\sqrt{3}} \cos \frac{61\pi}{6} &= \sqrt{2} \cos\left(4\pi + \frac{\pi}{4}\right) - \frac{1}{\sqrt{3}} \cos\left(10\pi + \frac{\pi}{6}\right) = \\ = \sqrt{2} \cos \frac{\pi}{4} - \frac{1}{\sqrt{3}} \cos \frac{\pi}{6} &= \sqrt{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = 1 - \frac{1}{2} = \frac{1}{2}; \\ 5) \frac{\sin(-6,5\pi) + \operatorname{tg}(-7\pi)}{\cos(-7\pi) + \operatorname{ctg}(-16,25\pi)} &= \frac{-\sin\left(6\pi + \frac{\pi}{2}\right) - \operatorname{tg}(6\pi + \pi)}{\cos(6\pi + \pi) - \operatorname{ctg}\left(16\pi + \frac{\pi}{4}\right)} = \\ = \frac{-\sin \frac{\pi}{2} - \operatorname{tg} \pi}{\cos \pi - \operatorname{ctg} \frac{\pi}{4}} &= \frac{-1 - 0}{-1 - 1} = \frac{1}{2}; \\ 6) \frac{\cos(-540^\circ) + \sin 480^\circ}{\operatorname{tg} 405^\circ - \operatorname{ctg} 330^\circ} &= \frac{\cos(720^\circ - 180^\circ) + \sin(360^\circ + 120^\circ)}{\operatorname{tg}(360^\circ + 45^\circ) - \operatorname{ctg}(360^\circ - 30^\circ)} = \\ = \frac{\cos 180^\circ + \sin 120^\circ}{\operatorname{tg} 45^\circ + \operatorname{ctg} 30^\circ} &= \frac{-1 + \frac{\sqrt{3}}{2}}{1 + \sqrt{3}} = \frac{\sqrt{3} - 2}{2(1 + \sqrt{3})} = \frac{(\sqrt{3} - 2)(1 - \sqrt{3})}{2(1 + \sqrt{3})(1 - \sqrt{3})} = \frac{5 - 3\sqrt{3}}{4}. \end{aligned}$$

330.

$$1) \frac{\sin\left(\frac{\pi}{2} - \alpha\right) + \sin(\pi - \alpha)}{\cos(\pi - \alpha) + \sin(2\pi - \alpha)} = \frac{\cos\alpha + \sin\alpha}{-\cos\alpha - \sin\alpha} = -1;$$

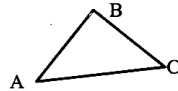
$$2) \frac{\cos(\pi - \alpha) + \cos\left(\frac{\pi}{2} - \alpha\right)}{\sin(\pi - \alpha) - \sin\left(\frac{\pi}{2} - \alpha\right)} = \frac{-\cos\alpha + \sin\alpha}{\sin\alpha - \cos\alpha} = 1;$$

$$3) \frac{\sin(\alpha - \pi) \cdot \operatorname{tg}(\pi - \alpha)}{\operatorname{tg}(\alpha + \pi) \cdot \cos\left(\frac{\pi}{2} - \alpha\right)} = \frac{-\sin\alpha \cdot (-\operatorname{tg}\alpha)}{\operatorname{tg}\alpha \cdot \sin\alpha} = 1;$$

$$4) \frac{\sin^2(\pi - \alpha) + \sin\left(\frac{\pi}{2} - \alpha\right)}{\sin(\pi - \alpha)} \cdot \operatorname{tg}(\pi - \alpha) = \frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha} \cdot (-\operatorname{tg}\alpha) = \\ = \frac{1}{\sin\alpha} \cdot \left(-\frac{\sin\alpha}{\cos\alpha}\right) = -\frac{1}{\cos\alpha}.$$

331.

Пусть α, β, γ – углы треугольника,



$$\sin\gamma = \sin(180^\circ - (\alpha + \beta)) = \sin 180^\circ \cdot \cos(\alpha + \beta) - \\ - \cos 180^\circ \cdot \sin(\alpha + \beta) = 0 \cdot \cos(\alpha + \beta) - (-1) \cdot \sin(\alpha + \beta) = \sin(\alpha + \beta).$$

332.

$$1) \sin\left(\frac{\pi}{2} + \alpha\right) = \sin\frac{\pi}{2} \cdot \cos\alpha + \cos\frac{\pi}{2} \cdot \sin\alpha = \\ = 1 \cdot \cos\alpha + 0 \cdot \sin\alpha = \cos\alpha;$$

$$2) \cos\left(\frac{\pi}{2} + \alpha\right) = \cos\frac{\pi}{2} \cdot \cos\alpha - \sin\frac{\pi}{2} \cdot \sin\alpha = \\ = 0 \cdot \cos\alpha - 1 \cdot \sin\alpha = -\sin\alpha;$$

$$3) \cos\left(\frac{3\pi}{2} - \alpha\right) = \cos\frac{3\pi}{2} \cdot \cos\alpha + \sin\frac{3\pi}{2} \cdot \sin\alpha = \\ = 0 \cdot \cos\alpha + (-1) \cdot \sin\alpha = -\sin\alpha;$$

$$4) \sin\left(\frac{3\pi}{2} - \alpha\right) = \sin\frac{3\pi}{2} \cdot \cos\alpha - \cos\frac{3\pi}{2} \cdot \sin\alpha = \\ = -1 \cdot \cos\alpha - 0 \cdot \sin\alpha = -\cos\alpha.$$

333.

$$1) \cos\left(\frac{\pi}{2} - x\right) = 1;$$

$$\sin x = 1.$$

$$\text{Тогда } x = \frac{\pi}{4} + 2\pi n, \quad n \in Z.$$

$$3) \cos(x - \pi) = 0;$$

$$\cos x = 0.$$

$$\text{Поэтому } x = \frac{\pi}{4} + 2\pi n, \quad n \in Z.$$

$$2) \sin(\pi - x) = 1;$$

$$\sin x = 1.$$

$$\text{Значит } x = \frac{\pi}{4} + 2\pi n, \quad n \in Z.$$

$$4) \sin\left(x - \frac{\pi}{2}\right) = 1;$$

$$-\cos x = 1;$$

$$\cos x = -1.$$

$$\text{Тогда } x = \pi + 2\pi n, \quad n \in Z.$$

334.

$$1) \sin\left(\frac{\pi}{4} + \alpha\right) - \cos\left(\frac{\pi}{4} - \alpha\right) = \\ = \frac{\sqrt{2}}{2} \cos \alpha + \frac{\sqrt{2}}{2} \sin \alpha - \frac{\sqrt{2}}{2} \cos \alpha - \frac{\sqrt{2}}{2} \sin \alpha = 0;$$

$$2) \cos\left(\frac{\pi}{6} - \alpha\right) - \sin\left(\frac{\pi}{3} + \alpha\right) = \\ = \frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha - \frac{\sqrt{3}}{2} \cos \alpha - \frac{1}{2} \sin \alpha = 0.$$

336.

1) I четв.;

3) III четв.;

5) II четв.

2) III четв.;

4) IV четв.;

337.

$$1) \sin 3\pi = 0; \cos 3\pi = -1;$$

$$3) \sin 3,5\pi = -1;$$

$$\cos 3,5\pi = 0;$$

$$5) \sin \pi n = 0;$$

$$\cos \pi n = \begin{cases} 1, & n - \text{четное} \\ -1, & n - \text{нечетное} \end{cases};$$

$$2) \sin 4\pi = 0; \cos 4\pi = 1;$$

$$4) \sin\left(\frac{5\pi}{2}\right) = \sin \frac{\pi}{2} = 1;$$

$$\cos \frac{5\pi}{2} = 0;$$

$$6) \sin((2n+1)\pi) = 0;$$

$$\cos((2n+1)\pi) = -1, \quad n \in Z.$$

338.

$$1) \sin 3\pi - \cos \frac{3\pi}{2} = 0 - 0 = 0;$$

$$2) \cos 0 - \cos 3\pi + \cos 3,5\pi = 1 - (-1) + 0 = 2;$$

$$3) \sin \pi k + \cos 2\pi k = 0 + 1 = 1;$$

$$4) \cos \frac{(2k+1)\pi}{2} - \sin \frac{(4k+1)\pi}{2} = 0 - 1 = -1.$$

339.

1) Т.к. $\frac{\pi}{2} < \alpha < \pi$, то $\cos \alpha < 0$, тогда

$$\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \frac{1}{9}} = -\frac{\sqrt{6}}{3}.$$

2) Т.к. $\pi < \alpha < \frac{3\pi}{2}$, то $\operatorname{tg} \alpha < 0$,

$$\text{т.к. } 1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}, \text{ то } \operatorname{tg} \alpha = \sqrt{\frac{1}{\cos^2 \alpha} - 1} = \sqrt{\frac{9}{5} - 1} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}.$$

3) Т.к. $0 < \alpha < \frac{\pi}{2}$, то $\sin \alpha > 0$, $\operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$;

$$1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha} \quad \sin \alpha = \sqrt{\frac{1}{1 + \operatorname{ctg}^2 \alpha}} = \sqrt{\frac{1}{1 + \frac{1}{8}}} = \sqrt{\frac{8}{9}} = \sqrt{\frac{2\sqrt{2}}{3}}.$$

4) Т.к. $\pi < \alpha < \frac{3\pi}{2}$, то $\cos \alpha < 0$,

$$1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}, \quad \operatorname{tg} \alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2};$$

$$\cos \alpha = -\sqrt{\frac{1}{1 + \operatorname{tg}^2 \alpha}} = \sqrt{\frac{1}{1 + \frac{1}{2}}} = -\sqrt{\frac{2}{3}} = \frac{-\sqrt{6}}{3}.$$

340.

$$1) 5\sin^2 \alpha + \operatorname{tg} \alpha \cdot \cos \alpha + 5\cos^2 \alpha = \\ = 5(\sin^2 \alpha + \cos^2 \alpha) + \frac{\sin \alpha}{\cos \alpha} \cdot \cos \alpha = 5 + \sin \alpha;$$

$$2) \operatorname{ctg} \alpha \cdot \sin \alpha - 2\cos^2 \alpha - 2\sin^2 \alpha = \\ = \frac{\sin \alpha}{\cos \alpha} \cdot \sin \alpha - 2(\sin^2 \alpha + \cos^2 \alpha) = \cos \alpha - 2;$$

$$3) \frac{3}{1 + \operatorname{tg}^2 \alpha} = 3 \cos^2 \alpha. \text{ Т.к. } \cos^2 \alpha = \frac{1}{1 + \operatorname{tg}^2 \alpha};$$

$$4) \frac{5}{1 + \operatorname{ctg}^2 \alpha} = 5 \sin^2 \alpha. \text{ Т.к. } \sin^2 \alpha = \frac{1}{1 + \operatorname{ctg}^2 \alpha}.$$

341.

$$\begin{aligned} 1) & 2\sin(-\alpha) \cdot \cos\left(\frac{\pi}{2} - \alpha\right) - 2\cos(-\alpha) \cdot \sin\left(\frac{\pi}{2} - \alpha\right) = \\ & = -2\sin\alpha \cdot \sin\alpha - 2\cos\alpha \cdot \cos\alpha = -2\sin^2\alpha - 2\cos^2\alpha = \\ & = -2(\sin^2\alpha + \cos^2\alpha) = -2; \end{aligned}$$

$$\begin{aligned} 2) & 3\sin(\pi - \alpha)\cos\left(\frac{\pi}{2} - \alpha\right) + 3\sin^2\left(\frac{\pi}{2} - \alpha\right) = \\ & = 3\sin\alpha \cdot \sin\alpha + 3\cos^2\alpha = 3(\sin^2\alpha + \cos^2\alpha) = 3; \end{aligned}$$

$$\begin{aligned} 3) & (1 - \operatorname{tg}(-\alpha)) \cdot (1 - \operatorname{tg}(\pi + \alpha))\cos^2\alpha = (1 + \operatorname{tg}\alpha)(1 - \operatorname{tg}\alpha) \cdot \cos^2\alpha = \\ & = (1 - \operatorname{tg}^2\alpha) \cdot \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha; \end{aligned}$$

$$\begin{aligned} 4) & (1 + \operatorname{tg}^2(-\alpha)) \cdot \left(\frac{1}{1 + \operatorname{tg}^2(-\alpha)}\right) = (1 + \operatorname{tg}^2\alpha) \cdot \frac{1}{1 + \operatorname{tg}^2\alpha} = \\ & = \frac{(1 + \operatorname{tg}^2\alpha) \cdot \operatorname{tg}^2\alpha}{1 + \operatorname{tg}^2\alpha} = \operatorname{tg}^2\alpha. \end{aligned}$$

342.

$$1) \sin\left(\frac{3\pi}{2} - \alpha\right) + \sin\left(\frac{3\pi}{2} + \alpha\right) = \cos\alpha - \cos\alpha = -2\cos\alpha.$$

$$\text{Т.к. } \cos\alpha = \frac{1}{4}, \text{ то значение выражения равно } -\frac{1}{2}.$$

$$2) \cos\left(\frac{\pi}{2} + \alpha\right) + \cos\left(\frac{3\pi}{2} - \alpha\right) = -\sin\alpha - \sin\alpha = -2\sin\alpha.$$

$$\text{Т.к. } \sin\alpha = \frac{1}{6}, \text{ то значение выражения равно } -\frac{1}{3}.$$

343.

$$1) 2\sin 75^\circ \cdot \cos 75^\circ = \sin 150^\circ = \sin(180^\circ - 150^\circ) = \sin 30^\circ = \frac{1}{2};$$

$$\begin{aligned} 2) & \cos^2 75^\circ - \sin^2 75^\circ = \cos 150^\circ = -\cos(180^\circ - 150^\circ) = \\ & = -\cos 30^\circ = -\frac{\sqrt{3}}{2}; \end{aligned}$$

$$3) \sin 15^\circ = \sin(45^\circ - 30^\circ) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}(\sqrt{3}-1)}{4} = \frac{\sqrt{6}-\sqrt{2}}{4};$$

$$4) \sin 75^\circ = \sin(45^\circ + 30^\circ) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}(\sqrt{3}+1)}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}.$$

344.

$$1) \cos^2(\pi - \alpha) - \cos^2\left(\frac{\pi}{2} - \alpha\right) = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha;$$

$$2) 2\sin\left(\frac{\pi}{2} - \alpha\right)\cos\left(\frac{\pi}{2} - \alpha\right) = 2 \cdot \cos\alpha \cdot \sin\alpha = \sin 2\alpha;$$

$$3) \frac{\cos^2(2\pi + \alpha) - \sin^2(2\pi + \alpha)}{2\cos(2\pi + \alpha)\cos\left(\frac{\pi}{2} - \alpha\right)} = \frac{\cos^2\alpha - \sin^2\alpha}{2\cos\alpha\sin\alpha} = \frac{\cos 2\alpha}{\sin 2\alpha} = \operatorname{ctg} 2\alpha;$$

$$4) \frac{2\sin(\pi - \alpha)\sin\left(\frac{\pi}{2} - \alpha\right)}{\sin^2\left(\alpha - \frac{\pi}{2}\right) - \sin^2(\alpha - \pi)} = \frac{2\cos\alpha\sin\alpha}{\cos^2\alpha - \sin^2\alpha} = \frac{\sin 2\alpha}{\cos 2\alpha} = \operatorname{tg} 2\alpha.$$

345.

$$1) \sin \frac{47\pi}{6} = \sin\left(8\pi - \frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2};$$

$$2) \operatorname{tg} \frac{25\pi}{4} = \operatorname{tg}\left(6\pi + \frac{\pi}{4}\right) = \operatorname{tg} \frac{\pi}{4} = 1;$$

$$3) \operatorname{ctg} \frac{27\pi}{4} = \operatorname{ctg}\left(7\pi - \frac{\pi}{4}\right) = \operatorname{ctg}\left(-\frac{\pi}{4}\right) = \operatorname{ctg}\left(-\frac{\pi}{4}\right) = -1;$$

$$4) \cos \frac{21\pi}{4} = \cos\left(5\pi + \frac{\pi}{4}\right) = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}.$$

346.

$$1) \cos \frac{23\pi}{4} - \sin \frac{15\pi}{4} = \cos\left(6\pi - \frac{\pi}{4}\right) - \sin\left(4\pi + \frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right) = \\ = \cos \frac{\pi}{4} + \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2};$$

$$2) \sin \frac{25\pi}{3} - \operatorname{tg} \frac{10\pi}{3} = \sin\left(8\pi + \frac{\pi}{3}\right) - \operatorname{tg}\left(3\pi + \frac{\pi}{3}\right) = \sin \frac{\pi}{3} - \operatorname{tg} \frac{\pi}{3} = \\ = \frac{\sqrt{3}}{2} - \sqrt{3} = -\frac{\sqrt{3}}{2};$$

$$\begin{aligned}
 & 3) 3\cos 360^\circ + \sin(-1560^\circ) = 3\cos(10 \cdot 360^\circ + 60^\circ) + \\
 & + \sin(-120^\circ - 4 \cdot 360^\circ) = 3 \cdot \cos 60^\circ - \sin 120^\circ = 3 \cdot \frac{1}{2} - \sin 60^\circ = \\
 & = \frac{3}{2} - \frac{\sqrt{3}}{2} = \frac{3 - \sqrt{3}}{2};
 \end{aligned}$$

$$\begin{aligned}
 & 4) \cos(-945^\circ) + \operatorname{tg} 1035^\circ = \cos(-3 \cdot 360^\circ + 135^\circ) + \\
 & + \operatorname{tg}(2,5 \cdot 360^\circ + 135^\circ) = \cos 135^\circ + \operatorname{tg} 135^\circ = -\cos 45^\circ - \operatorname{tg} 45^\circ = \\
 & = -\frac{\sqrt{2}}{2} - 1 = -\frac{2 + \sqrt{2}}{2}.
 \end{aligned}$$

347.

1) $\sin 3 > \cos 4$,

2) $\cos 0 > \sin 5$,

т.к. $\sin 3 > 0$, $\cos 4 < 0$.

т.к. $\sin 5 < 0$, $\cos 0 = 1$.

348.

1) $\sin 3,5 \cdot \operatorname{tg} 3,5 = \frac{\sin^2 3,5}{\cos 3,5} < 0$, т.к. $\sin^2 3,5 > 0$, $\cos 3,5 < 0$;

2) $\cos 5,01 \cdot \sin 0,73 > 0$, т.к. $\cos 5,01 > 0$, $\sin 0,73 > 0$;

3) $\frac{\operatorname{tg} 13}{\cos 15} < 0$, т.к. $\operatorname{tg} 13 > 0$, $\cos 15 < 0$;

4) $\sin 1 \cdot \cos 2 \cdot \operatorname{tg} 3 > 0$, т.к. $\sin 1 > 0$, $\cos 2$ и $\operatorname{tg} 3 < 0$.

349.

1) $\sin \frac{\pi}{8} \cdot \cos \frac{3\pi}{8} + \sin \frac{3\pi}{8} \cdot \cos \frac{\pi}{8} = \sin\left(\frac{\pi}{8} + \frac{3\pi}{8}\right) = \sin \frac{\pi}{2} = 1$;

2) $\sin 165^\circ = \sin(120^\circ + 45^\circ) = \sin 120^\circ \cdot \cos 45^\circ + \cos 120^\circ \cdot \sin 45^\circ =$
 $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}(\sqrt{3} - 1)}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$;

3) $\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ =$
 $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}(\sqrt{3} + 1)}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$;

4) $\sin \frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin \frac{\pi}{3} \cdot \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \cdot \sin \frac{\pi}{4} =$
 $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}(\sqrt{3} - 1)}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$;

5) $1 - \sin^2 195^\circ = \cos^2 195^\circ - \sin^2 195^\circ = \cos 390^\circ = \cos(360^\circ + 30^\circ) =$
 $= \cos 30^\circ = \frac{\sqrt{3}}{2}$;

$$\begin{aligned}
 & 5) 2 \cos^2 \frac{3\pi}{8} - 1 = 2 \cos^2 \frac{3\pi}{8} - \cos^2 \frac{3\pi}{8} - \sin^2 \frac{3\pi}{8} = \cos^2 \frac{3\pi}{8} - \sin^2 \frac{3\pi}{8} = \\
 & = \cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2};
 \end{aligned}$$

350.

$$1) (1 + \operatorname{tg}(-\alpha)) \cdot (1 - \operatorname{ctg}(-\alpha)) - \frac{\sin(-\alpha)}{\cos(-\alpha)} = (1 - \operatorname{tg}\alpha) \cdot (1 + \operatorname{ctg}\alpha) +$$

$$+ \operatorname{tg}\alpha = 1 + \operatorname{ctg}\alpha - \operatorname{tg}\alpha - 1 + \operatorname{tg}\alpha = \operatorname{ctg}\alpha;$$

$$\begin{aligned}
 2) \frac{\operatorname{ctg}\alpha + \operatorname{tg}(-\alpha)}{\cos\alpha + \sin(-\alpha)} + \frac{\operatorname{tg}(-\alpha)}{\sin\alpha} &= \frac{\operatorname{ctg}\alpha - \operatorname{tg}\alpha}{\cos\alpha - \sin\alpha} - \frac{1}{\cos\alpha} = \\
 &= \frac{\cos^2\alpha - \sin^2\alpha}{\cos\alpha \cdot \sin\alpha(\cos\alpha - \sin\alpha)} - \frac{1}{\cos\alpha} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha \cdot \sin\alpha} - \frac{1}{\cos\alpha} = \\
 &= \frac{\cos\alpha}{\cos\alpha \cdot \sin\alpha} = \frac{1}{\sin\alpha}.
 \end{aligned}$$

351.

$$\text{Т.к. } \frac{\pi}{2} < \alpha < \pi, \text{ то } \cos\alpha < 0, \text{ тогда } \cos\alpha = -\sqrt{1 - \sin^2\alpha} = -\sqrt{1 - \frac{5}{9}} = -\frac{2}{3};$$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = -\frac{\sqrt{5}}{2}; \operatorname{ctg}\alpha = \frac{1}{\operatorname{tg}\alpha} = -\frac{2}{\sqrt{5}};$$

$$\sin 2\alpha = 2\sin\alpha \cos\alpha = 2 \cdot \frac{\sqrt{5}}{3} \cdot \left(-\frac{2}{3}\right) = -\frac{4\sqrt{5}}{9};$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \frac{4}{9} - \frac{5}{9} = -\frac{1}{9};$$

352.

$$\begin{aligned}
 1) \cos^3\alpha \cdot \sin\alpha - \sin^3\alpha \cdot \cos\alpha &= \cos\alpha \cdot \sin\alpha(\cos^2\alpha - \sin^2\alpha) = \\
 &= \frac{1}{2} \sin 2\alpha \cdot \cos 2\alpha = \frac{1}{4} \sin 4\alpha;
 \end{aligned}$$

$$2) \frac{\sin\alpha + \sin 2\alpha}{1 + \cos\alpha + \cos 2\alpha} = \frac{\sin\alpha(1 + 2\cos\alpha)}{2\cos^2\alpha + \cos\alpha} = \frac{\sin\alpha(1 + 2\cos\alpha)}{\cos\alpha(1 + 2\cos\alpha)} = \operatorname{tg}\alpha.$$

353.

$$1) \frac{\sin 2\alpha - \sin 2\alpha \cdot \cos 2\alpha}{4\cos\alpha} = \frac{\sin 2\alpha(1 - \cos 2\alpha)}{4\cos\alpha} = \frac{2\sin\alpha \cos\alpha \cdot 2\sin^2\alpha}{4\cos\alpha} = \sin^3\alpha;$$

$$2) \frac{2 \cos^2 2\alpha}{\sin 4\alpha \cdot \cos 4\alpha + \sin 4\alpha} = \frac{2 \cos^2 2\alpha}{\sin 4\alpha(\cos 4\alpha + 1)} =$$

$$= \frac{2 \cos^2 2\alpha}{2 \sin 2\alpha \cos 2\alpha(2 \cos^2 2\alpha)} = \frac{1}{2 \sin 2\alpha \cos 2\alpha} = \frac{1}{\sin 4\alpha};$$

$$3) \frac{\cos 2\alpha + \sin 2\alpha \cdot \cos 2\alpha}{2 \sin^2 \alpha - 1} = \frac{\cos 2\alpha(1 + \sin 2\alpha)}{\sin^2 \alpha - \cos^2 \alpha} =$$

$$= \frac{\cos 2\alpha(1 + \sin 2\alpha)}{-\cos 2\alpha} = -(1 + \sin 2\alpha);$$

$$4) \frac{(\cos \alpha - \sin \alpha)^2}{\sin 2\alpha \cdot \cos 2\alpha - \cos 2\alpha} = \frac{1 - 2 \cos \alpha \sin \alpha}{\cos 2\alpha(\sin 2\alpha - 1)} =$$

$$= \frac{-(\sin 2\alpha - 1)}{\cos 2\alpha(\sin 2\alpha - 1)} = \frac{-1}{\cos 2\alpha}.$$

354.

$$1) \frac{\cos^2 x}{1 - \sin x} - \sin(\pi - x) = \frac{\cos^2 x - \sin x(1 - \sin x)}{(1 - \sin x)} = \frac{1 - \sin x}{1 - \sin x} = 1;$$

$$2) \frac{\cos^2 x}{1 + \sin x} + \cos(1,5\pi + x) = \frac{\cos^2 x + \sin x(1 + \sin x)}{1 + \sin x} = \frac{1 + \sin x}{1 + \sin x} = 1;$$

$$3) \frac{\sin^2 x}{1 + \cos x} - \sin(1,5\pi + x) = \frac{\sin^2 x + \cos x(1 - \cos x)}{1 + \cos x} = \frac{1 + \cos x}{1 + \cos x} = 1;$$

$$4) \frac{\sin^2 x}{1 - \cos x} + \cos(3\pi - x) = \frac{\sin^2 x - \cos x(1 - \cos x)}{1 - \cos x} = \frac{1 - \cos x}{1 - \cos x} = 1.$$

355.

$$1) \operatorname{tg} \alpha + \operatorname{ctg} \alpha = \operatorname{tg} \alpha + \frac{1}{\operatorname{tg} \alpha} = \frac{\operatorname{tg}^2 \alpha + 1}{\operatorname{tg} \alpha} = \frac{(\sin^2 \alpha + \cos^2 \alpha) \cos \alpha}{\cos^2 \alpha \cdot \sin \alpha} =$$

$$= \frac{1}{\cos \alpha \sin \alpha} = \frac{1}{\frac{1}{2} \sin 2\alpha} = \frac{2}{\sin 2\alpha}, \text{ т.к. } \alpha = -\frac{\pi}{12}, \text{ то}$$

$$\sin 2\alpha = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2} \text{ и значение выражения равно } \frac{2}{-\frac{1}{2}} = -4;$$

$$2) \operatorname{ctg} \alpha - \operatorname{tg} \alpha = \frac{\cos \alpha}{\sin \alpha} - \frac{\sin \alpha}{\cos \alpha} = \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos \alpha \cdot \sin \alpha} = \frac{\cos 2\alpha}{\frac{1}{2} \sin \alpha} = 2 \operatorname{ctg} 2\alpha.$$

$$\text{Т.к. } \alpha = -\frac{\pi}{8}, \text{ то } 2 \operatorname{ctg} 2\alpha = 2 \operatorname{ctg}\left(-\frac{\pi}{4}\right) = 2;$$

$$3) \frac{\cos \alpha}{\cos \alpha + \sin \alpha} + \frac{\sin \alpha}{\cos \alpha - \sin \alpha} =$$

$$= \frac{\cos^2 \alpha - \cos \alpha \cdot \sin \alpha + \cos \alpha \cdot \sin \alpha + \sin^2 \alpha}{\cos^2 \alpha - \sin^2 \alpha} = \frac{1}{\cos 2\alpha}.$$

Т.к. $\alpha = -\frac{\pi}{6}$, то $\frac{1}{\cos 2\alpha} = \frac{1}{\cos\left(-\frac{\pi}{3}\right)} = \frac{1}{1/2} = 2$;

$$4) \frac{\sin \alpha}{\cos \alpha + \sin \alpha} - \frac{\cos \alpha}{\cos \alpha - \sin \alpha} =$$

$$= \frac{\sin \alpha \cdot \cos \alpha - \sin^2 \alpha - \cos^2 \alpha - \cos \alpha \cdot \sin \alpha}{\cos^2 \alpha - \sin^2 \alpha} = \frac{-1}{\cos 2\alpha}.$$

Т.к. $\alpha = \frac{\pi}{3}$, то $\frac{1}{\cos 2\alpha} = \frac{-1}{\cos \frac{2\pi}{3}} = \frac{-1}{-1/2} = 2$.

356.

$$2 \sin\left(\frac{\pi}{2} + \alpha\right) \sin(\pi - \alpha) + \cos 2\alpha - 1$$

$$\frac{\cos 2\alpha + \sin \alpha \cdot \cos \alpha - \cos^2 \alpha}{\cos^2 \alpha - \sin^2 \alpha + \sin \alpha \cdot \cos \alpha - \cos^2 \alpha} =$$

$$= \frac{2 \cos \alpha \sin \alpha + \cos^2 \alpha - \sin^2 \alpha - \cos^2 \alpha - \sin^2 \alpha}{\sin \alpha (\cos \alpha - \sin \alpha)} = \frac{2 \sin \alpha (\cos \alpha - \sin \alpha)}{\sin \alpha (\cos \alpha - \sin \alpha)} = 2$$

357.

$$1) \sin(2x + 3\pi) \sin\left(x + \frac{3\pi}{2}\right) - \sin 3x \cos 2x = -1;$$

$$-\sin 2x \cdot (-\cos 3x) - \sin 3x \cos 2x = -1; \sin(3x - 2x) = 1, \text{ т.е. } \sin x = 1.$$

Тогда $x = \frac{\pi}{2} + 2\pi n, \quad n \in \mathbb{Z}.$

$$2) \sin\left(5x - \frac{3\pi}{2}\right) \cdot \cos(2x + 4\pi) - \sin(5x + \pi) \sin 2x = 0;$$

$$\left. \begin{array}{l} \cos 5x \cdot \cos 2x + \sin 5x \sin 2x = 0; \\ \cos(5x - 2x) = 1; \\ \cos 3x = 0. \end{array} \right\} \begin{array}{l} \text{Тогда } 3x = \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z} \\ \\ u \quad x = \frac{\pi}{6} + \frac{\pi n}{3}, \quad n \in \mathbb{Z}. \end{array}$$

358.

$$1) \operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \operatorname{tg}\beta}, \text{ т.к. } \operatorname{tg}\alpha = -\frac{3}{4}, \operatorname{tg}\beta = 2,4, \text{ то}$$

$$\operatorname{tg}(\alpha + \beta) = \frac{\frac{3}{4} + 2,4}{1 + \frac{3}{4} \cdot 2,4} = \frac{1,65}{2,8} = \frac{165}{280} = \frac{33}{56};$$

$$2) \operatorname{ctg}(\alpha + \beta) = \frac{1}{\operatorname{tg}(\alpha + \beta)}. \text{ Т.к. } \operatorname{ctg}\alpha = \frac{4}{3}, \text{ то } \operatorname{tg}\alpha = \frac{3}{4},$$

$$\begin{aligned} \text{т.к. } \operatorname{ctg}\beta = -1, \text{ то } \operatorname{tg}\beta = -1; \operatorname{tg}(\alpha + \beta) &= \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \cdot \operatorname{tg}\beta} = \\ &= \frac{\frac{3}{4} - 1}{1 - \frac{3}{4} \cdot (-1)} = \frac{-\frac{1}{4}}{1\frac{3}{4}} = -\frac{1}{7}, \text{ поэтому } \operatorname{ctg}(\alpha + \beta) = -7. \end{aligned}$$

359.

$$\begin{aligned} 1) 2 \sin\left(\frac{\pi}{4} + 2\alpha\right) \sin\left(\frac{\pi}{4} - 2\alpha\right) &= 2 \sin\left(\frac{\pi}{4} + 2\alpha\right) \sin\left(\frac{\pi}{2} - \left(\frac{\pi}{4} + 2\alpha\right)\right) = \\ &= 2 \sin\left(\frac{\pi}{4} + 2\alpha\right) \cos\left(\frac{\pi}{4} + 2\alpha\right) = \sin\left(\frac{\pi}{2} + 4\alpha\right) = \cos 4\alpha; \end{aligned}$$

$$\begin{aligned} 2) 2 \cos\left(\frac{\pi}{4} + 2\alpha\right) \cdot \cos\left(\frac{\pi}{4} - 2\alpha\right) &= 2 \cos\left(\frac{\pi}{4} + 2\alpha\right) \cos\left(\frac{\pi}{2} - \left(\frac{\pi}{4} + 2\alpha\right)\right) = \\ &= 2 \sin\left(\frac{\pi}{4} + 2\alpha\right) \cos\left(\frac{\pi}{4} + 2\alpha\right) = \sin\left(\frac{\pi}{2} + 4\alpha\right) = \cos 4\alpha; \end{aligned}$$

$$\begin{aligned} 3) \cos^2\left(\frac{\pi}{4} - \alpha\right) - \cos^2\left(\frac{\pi}{4} + \alpha\right) &= \cos^2\left(\frac{\pi}{2} - \left(\frac{\pi}{4} + \alpha\right)\right) - \cos^2\left(\frac{\pi}{4} + \alpha\right) = \\ &= \sin^2\left(\frac{\pi}{4} + \alpha\right) - \cos^2\left(\frac{\pi}{4} + \alpha\right) = -\cos^2\left(\frac{\pi}{2} + 2\alpha\right) = \sin 2\alpha; \end{aligned}$$

$$\begin{aligned} 4) \sin^2\left(\frac{\pi}{4} + \alpha\right) - \sin^2\left(\frac{\pi}{4} - \alpha\right) &= \sin^2\left(\frac{\pi}{2} - \left(\frac{\pi}{4} - \alpha\right)\right) - \sin^2\left(\frac{\pi}{4} - \alpha\right) = \\ &= \cos^2\left(\frac{\pi}{4} - \alpha\right) - \sin^2\left(\frac{\pi}{4} - \alpha\right) = \cos\left(\frac{\pi}{2} - 2\alpha\right) = \sin 2\alpha. \end{aligned}$$

360.

$$1) 1 + \cos 2x = 2 \cos x; \\ 2 \cos^2 x - 2 \cos x = 0;$$

$$2) 1 - \cos 2x = 2 \sin x; \\ 2 \sin^2 x - 2 \sin x = 0;$$

$$2\cos x (\cos x - 1) = 0; \begin{cases} \cos x = 0, \\ \cos x = 1 \end{cases}; \quad 2\sin x (\sin x - 1) = 0; \begin{cases} \sin x = 0, \\ \sin x = 1 \end{cases};$$

$$\begin{cases} x = \frac{\pi}{2} + \pi n, n \in Z; \\ x = 2\pi k, k \in Z \end{cases}; \quad \begin{cases} x = \pi n, n \in Z \\ x = \frac{\pi}{2} + 2\pi k, k \in Z \end{cases}.$$

Глава V. Прогрессия

361.

- 1) $a_3 = 9$; $a_6 = 36$, $a_n = n^2$;
 2) $a_k = 4$, если $k = 2$; $a_k = 25$, если $k = 5$;
 $a_k = n^2$, если $k = n$; $a_k = (n + 1)^2$, если $k = n + 1$.

362.

1) Пусть $\underline{a_n = 2n + 3}$;

$$a_1 = 2 \cdot 1 + 3 = 5;$$

$$a_2 = 2 \cdot 2 + 3 = 7;$$

$$a_3 = 2 \cdot 3 + 3 = 9.$$

2) Пусть $\underline{a_n = 1 + 3n}$;

$$a_1 = 1 + 3 \cdot 1 = 4;$$

$$a_2 = 1 + 3 \cdot 2 = 7;$$

$$a_3 = 1 + 3 \cdot 3 = 10.$$

3) Пусть $\underline{a_n = 100 - 10n^2}$;

$$a_1 = 100 - 10 \cdot 1 = 100 - 10 = 90;$$

$$a_2 = 100 - 10 \cdot 4 = 100 - 40 = 60;$$

$$a_3 = 100 - 10 \cdot 9 = 100 - 90 = 10.$$

4) Пусть $\underline{a_n = \frac{n-2}{3}}$;

$$a_1 = \frac{1-2}{3} = -\frac{1}{3};$$

$$a_2 = \frac{2-2}{3} = 0;$$

$$a_3 = \frac{3-2}{3} = \frac{1}{3}.$$

5) Пусть $\underline{a_n = \frac{1}{n}}$;

$$a_1 = 1; a_2 = \frac{1}{2}; a_3 = \frac{1}{3}.$$

6) Пусть $\underline{a_n = -n^3}$;

$$a_1 = -1; a_2 = -8; a_3 = -27.$$

363.

$$x_n = n^2$$

если $x_n = 100$, то $n = 10$; если $x_n = 144$, то $n = 12$

если $x_n = 225$, то $n = 15$

49, 169 – члены последовательности $x_n = n^2$, т.к. $49 = 7^2$, $169 = 13^2$

48 – не члены последовательности $x_n = n^2$.

364.

1) пусть $a_n = -3$, тогда $-3 = n^2 - 2n - 6$;

$n^2 - 2n - 3 = 0$. Решим: $n_1 = 3$; $n_2 = -1$ – не подходит, т.к. $n \in \mathbb{N}$;

$\boxed{a_3 = -3}$ – член a_n ;

2) пусть $a_n = 2$, тогда $2 = n^2 - 2n - 6$;

$n^2 - 2n - 8 = 0$. Решим: $n_1 = 4$; $n_2 = -2$ – не подходит, т.к. $n \in \mathbb{N}$;

$\boxed{a_4 = 2}$ – член a_n ;

3) пусть $a_n = 3$, тогда $3 = n^2 - 2n - 6$;

$n^2 - 2n - 9 = 0$. Решим: $\frac{D}{4} = 1 + 9 = 10$;

$n_{1,2} = \frac{1 \pm \sqrt{10}}{1}$ – не подходят, т.к. $n \in \mathbb{N}$;

$a_n = n^2 - 2n - 6$ $a_n = -3$ – не член a_n ;

4) пусть $a_n = 9$, тогда $9 = n^2 - 2n - 6$;

$n^2 - 2n - 15 = 0$. Решим: $n_1 = 5$; $n_2 = -3$ – не подходит, т.к. $n \in \mathbb{N}$;

$\boxed{a_5 = 9}$ – член a_n .

365.

1) $a_2 = 3a_1 + 1 = 3 \cdot 2 + 1 = 6 + 1 = 7$;

$a_3 = 3a_2 + 1 = 3 \cdot 7 + 1 = 21 + 1 = 22$;

$a_4 = 3a_3 + 1 = 3 \cdot 22 + 1 = 66 + 1 = 67$;

2) $a_2 = 5 - 2a_1 = 5 - 2 \cdot 2 = 5 - 4 = 1$;

$a_3 = 5 - 2a_2 = 5 - 2 \cdot 1 = 5 - 2 = 3$;

$a_4 = 5 - 2a_3 = 5 - 2 \cdot 3 = 5 - 6 = -1$.

366.

1) Если $a_n = 150$, то

$$150 = (n-1)(n+4);$$

$$150 = n^2 + 3n - 4;$$

$n^2 + 3n - 154 = 0$. Решим:

$$D = 9 + 616 = 625 > 0,$$

$$n_{1,2} = \frac{-3 \pm 25}{2};$$

$$n_1 = 11, n_2 = -14 \notin \mathbb{N};$$

не подходит, т.к. $n \in \mathbb{N}$.

Ответ: $n = 11$.

2) Если $a_n = 104$, то

$$104 = (n-1)(n+4);$$

$$104 = n^2 + 3n - 4;$$

$n^2 + 3n - 108 = 0$. Решим:

$$D = 9 + 432 = 441 > 0,$$

$$n_{1,2} = \frac{-3 \pm 21}{2};$$

$$n_1 = 9, n_2 = -12 \notin \mathbb{N};$$

не подходит, т.к. $n \in \mathbb{N}$.

Ответ: $n = 9$.

367.

$$a_2 = \sqrt{a_1} = \sqrt{256} = \sqrt{16^2} = 16; a_3 = \sqrt{a_2} = \sqrt{16} = \sqrt{4^2} = 4;$$

$$a_4 = \sqrt{a_3} = \sqrt{4} = \sqrt{2^2} = 2.$$

368.

$$1) a_2 = \sin\left(\frac{\pi}{2} \cdot a_1\right) = \sin \frac{\pi}{2} = 1; \quad a_3 = \sin\left(\frac{\pi}{2} \cdot a_2\right) = \sin \frac{\pi}{2} = 1;$$

$$a_4 = \sin\left(\frac{\pi}{2} \cdot a_3\right) = \sin \frac{\pi}{2} = 1; \quad a_5 = \sin\left(\frac{\pi}{2} \cdot a_4\right) = \sin \frac{\pi}{2} = 1;$$

$$a_6 = \sin\left(\frac{\pi}{2} \cdot a_5\right) = \sin \frac{\pi}{2} = 1;$$

$$2) a_2 = \cos \pi = -1;$$

$$a_3 = \cos(-\pi) = -1;$$

$$a_4 = \cos \pi = -1;$$

$$a_5 = \cos(-\pi) = -1;$$

$$a_6 = \cos \pi = -1.$$

369.

$$a_3 = a_1^2 - a_2 = 2^2 - 3 = 1; \quad a_4 = a_2^2 - a_3 = 3^2 - 1 = 8;$$

$$a_5 = a_3^2 - a_4 = 1^2 - 8 = -7.$$

370.

1) Пусть $a_n = -5n + 4$;

$$a_{n+1} = -5(n+1) + 4 = -5n - 5 + 4;$$

$$\boxed{a_{n+1} = -5n - 1};$$

$$a_{n-1} = -5(n-1) + 4 = -5n + 5 + 4;$$

$$\boxed{a_{n-1} = -5n + 9};$$

$$a_{n+5} = -5(n+5) + 4 = -5n - 25 + 4;$$

$$\boxed{a_{n+5} = -5n - 21}.$$

2) Пусть $a_n = 2(n-10)$.

$$\text{Тогда } a_{n+1} = 2(n+1-10) = 2n+2-20;$$

$$a_{n+1} = 2n-18;$$

$$a_{n-1} = 2(n-1-10) = 2n-2-20;$$

$$a_{n-1} = 2n-22;$$

$$a_{n+5} = 2(n+5-10) = 2n+10-20;$$

$$a_{n+5} = 2n-10.$$

3) Пусть $a_n = 2 \cdot 3^{n+1}$. Тогда $a_{n+1} = 2 \cdot 3^{n+2}$;

$$a_{n-1} = 2 \cdot 3^n; \quad a_{n+5} = 2 \cdot 3^{n+6}.$$

4) Пусть $a_n = 7 \cdot \left(\frac{1}{2}\right)^{n+2}$.

$$\text{Тогда } a_{n+1} = 7 \cdot \left(\frac{1}{2}\right)^{n+3};$$

$$a_{n-1} = 7 \cdot \left(\frac{1}{2}\right)^{n+1}; \quad a_{n+5} = 7 \cdot \left(\frac{1}{2}\right)^{n+7}.$$

372.

1) Т.к. $a_n = a_1 + (n - 1)d$, то

$$a_2 = 2 + 5 = 7;$$

$$a_3 = 7 + 5 = 12;$$

$$a_4 = 12 + 5 = 17;$$

$$a_5 = 17 + 5 = 22;$$

2) Т.к. $a_2 = a_1 + d$, то

$$a_2 = -3 + 2 = -1;$$

$$a_3 = -1 + 2 = 1;$$

$$a_4 = 1 + 2 = 3;$$

$$a_5 = 3 + 2 = 5.$$

373.

1) $a_{n+1} = 3 - 4(n + 1)$;

$$a_{n+1} - a_n = 3 - 4(n + 1) - 3 + 4n = 3 - \underline{4n} - 4 - 3 + \underline{4n} = -4,$$

т.к. разность $a_{n+1} - a_n$ не зависит от n , то это – арифметическая прогрессия.

2) $a_{n+1} = -5 + 2(n + 1)$;

$$a_{n+1} - a_n = -5 + 2(n + 1) + 5 - 2n = -\underline{5} + \underline{2n} + 2 + \underline{5} - \underline{2n} = 2,$$

т.к. $a_{n+1} - a_n$ не зависит от n , то это – арифметическая прогрессия.

3) $a_{n+1} = 3(n + 2)$;

$$a_{n+1} - a_n = 3(n + 2) - 3(n + 1) = \underline{3n} + 6 - \underline{3n} - 3 = 3,$$

т.к. $a_{n+1} - a_n$ не зависит от n , то это – арифметическая прогрессия.

4) $a_{n+1} = 2(2 - n)$;

$$a_{n+1} - a_n = 2(2 - n) - 2(3 - n) = 4 - \underline{2n} - 6 + \underline{2n} = -2,$$

т.к. $a_{n+1} - a_n$ не зависит от n , то это – арифметическая прогрессия.

374.

1) $a_n = a_1 + (n - 1)d$, $n = 15$, поэтому

$$a_{15} = a_1 + 14d = 2 + 14 \cdot 3 = 2 + 42 = 44.$$

Ответ: $a_{15} = 44$.

2) $a_n = a_1 + (n - 1)d$, $n = 20$, тогда $a_{20} = a_1 + 19d$;

$$a_{20} = 3 + 19 \cdot 4 = 3 + 76 = 79.$$

Ответ: $a_{20} = 79$.

3) $a_n = a_1 + (n - 1)d$, $n = 18$, тогда $a_{18} = a_1 + 17d$;

$$a_{18} = -3 + 17 \cdot (-2) = -37.$$

Ответ: $a_{18} = -37$.

4) $a_n = a_1 + (n - 1)d$, $n = 11$, тогда $a_{11} = a_1 + 10d$;

$$a_{11} = -2 + 10 \cdot (-4) = -42.$$

Ответ: $a_{11} = -42$.

375.

1) $a_1 = 1$; $a_2 = 6$;

$$d = 6 - 1 = 5;$$

$$a_n = a_1 + (n - 1)d = 1 + (n - 1) \cdot 5;$$

$$a_n = 5n - 4;$$

2) $a_1 = 25$; $a_2 = 21$;

$$d = 21 - 25 = -4;$$

$$a_n = a_1 + (n - 1)d = 25 + (n - 1) \cdot (-4);$$

$$a_n = -4n + 29;$$

$$\begin{aligned}
 3) a_1 &= -4; a_2 = -6; \\
 d &= -6 - (-4) = -2; \\
 a_n &= a_1 + (n-1)d = \\
 &= -4 + (n-1) \cdot (-2); \\
 a_n &= -2n - 2;
 \end{aligned}$$

$$\begin{aligned}
 4) a_1 &= 1; a_2 = -4; \\
 d &= -4 - 1 = -5; \\
 a_n &= a_1 + (n-1)d = \\
 &= 1 + (n-1) \cdot (-5); \\
 a_n &= -5n + 6.
 \end{aligned}$$

376.

$$\begin{aligned}
 a_1 &= 44; d = 38 - 44 = -6; \\
 a_n &= a_1 + (n-1)d. \text{ Тогда } -22 = 44 + (n-1) \cdot (-6); \\
 0 &= 66 - 6n + 6; 6n = 50 + 22; \\
 6n &= 72; n = 12.
 \end{aligned}$$

377.

$$\begin{aligned}
 a_1 &= -18; a_2 = -15; d = -15 - (-18) = 3; \\
 a_n &= a_1 + (n-1)d. \\
 \text{Тогда } 12 &= -18 + (n-1) \cdot 3; \\
 30 &= 3n - 3; 3n = 33; n = 11. \\
 \text{Ответ: } 12 &\text{ является членом } a_n.
 \end{aligned}$$

378.

$$\begin{aligned}
 a_1 &= 1; a_2 = -5; d = -5 - 1 = -6; \\
 \text{Тогда } -59 &= 1 + (n-1) \cdot (-6); \\
 -60 &= -6n + 6; \\
 6n &= 66; \\
 n &= 11; \\
 a_{11} &= -59 \\
 \text{является членом } a_n.
 \end{aligned}$$

$$\begin{aligned}
 a_n &= a_1 + (n-1)d. \\
 \text{Значит } -46 &= 1 + (n-1) \cdot (-6); \\
 0 &= 47 - 6n + 6; \\
 6n &= 53; \\
 n &= 8\frac{5}{6} - \text{ не натуральное,} \\
 \text{значит, } -46 &\text{ не является} \\
 \text{членом } a_n.
 \end{aligned}$$

379.

$$\begin{aligned}
 1) a_n &= a_1 + (n-1)d; \\
 a_{16} &= a_1 + 15 \cdot d, \text{ т.к. } a_1 = 7, a_{16} = 67, \text{ то} \\
 67 &= 7 + 15d; 15d = 60. \text{ Отсюда } d = 4. \\
 2) a_9 &= a_1 + 8d, \text{ т.к. } a_1 = -4, a_9 = 0, \text{ то} \\
 0 &= -4 + 8d; 8d = 4. \text{ Тогда } d = \frac{1}{2}.
 \end{aligned}$$

380.

$$\begin{aligned}
 1) a_2 &= 12. \\
 \text{Т.к. } a_9 &= a_1 + 8 \cdot d, \text{ то} \\
 12 &= a_1 + 8 \cdot 1,5; \\
 a_1 &= 12 - 12; \\
 a_1 &= 0.
 \end{aligned}$$

$$\begin{aligned}
 2) a_7 &= -4. \\
 \text{Т.к. } a_7 &= a_1 + 6 \cdot d, \text{ то} \\
 -4 &= a_1 + 6 \cdot 1,5; \\
 a_1 &= -4 - 9; \\
 a_1 &= -13.
 \end{aligned}$$

381.

1) $\underline{d = -3; a_{11} = 20.}$

Т.к. $a_{11} = a_1 + 10d$, то

$$20 = a_1 + 10 \cdot (-3);$$

$$a_1 = 20 + 30 = 50;$$

$$a_1 = 50;$$

2) $\underline{a_{21} = -10; a_{22} = -5,5;}$

$d = a_{22} - a_{21} = -5,5 - (-10) = 4,5.$

Т.к. $a_{21} = a_1 + 20 \cdot d$, то

$$-10 = a_1 + 20 \cdot 4,5;$$

$$a_1 = -10 - 90 = -100.$$

382.

1) если $a_3 = 13; a_6 = 22.$

Т.к. $a_6 = a_3 + 3d$, то

$$22 = 13 + 3 \cdot d.$$

Тогда $3d = 9$

и $d = 3;$

$$a_3 = a_1 + 2d;$$

$$13 = a_1 + 2 \cdot 3;$$

$$a_1 = 13 - 6.$$

Получим $a_1 = 7.$

Значит $a_n = a_1 + (n - 1)d;$

$$a_n = 7 + (n - 1) \cdot 3.$$

Итак, $a_n = 3n + 4.$

2) если $a_2 = -7; a_7 = 18.$

Т.к. $a_7 = a_2 + 5d$, то

$$18 = -7 + 5d.$$

Значит $5d = 25$

и $d = 5;$

$$a_2 = a_1 + d;$$

$$a_1 = -7 - 5.$$

Получим $a_1 = -12.$

Значит $a_n = a_1 + (n - 1)d;$

$$a_n = -12 + (n - 1) \cdot 5.$$

Итак, $a_n = 5n - 17.$

383.

$a_1 = 15; a_2 = 13.$ Тогда $d = 13 - 15 = -2.$

Т.к. $a_n = a_1 + (n - 1)d$, то $a_n = 15 + (n - 1)(-2);$

$a_n = -2n + 17.$ Т.к. $a_n < 0$, то $-2n + 17 < 0; -2n < -17.$

Тогда $n > 8,5$, т.е. при $n \geq 9$ $a_n < 0.$

384.

Т.к. $a_n = a_1 + (n - 1)d$, то $a_n = -10 + (n - 1) \cdot \frac{1}{2};$

$a_n = \frac{1}{2}n - 10 \frac{1}{2}.$ Если $a_n < 2$, то $\frac{1}{2}n - 10 \frac{1}{2} < 2;$

$n - 21 < 4, n < 25.$ Т.е. при $n \leq 25; a_n < 2.$

385.

1) если $a_8 = 126, a_{10} = 146;$

$a_9 = \frac{a_8 + a_{10}}{2}$, тогда

$$a_9 = \frac{126 + 146}{2} = \frac{272}{2} = 136;$$

$d = a_9 - a_8,$

$$d = 136 - 126 = 10;$$

2) если $a_8 = -64, a_{10} = -50;$

$a_9 = \frac{a_8 + a_{10}}{2}$, тогда

$$a_9 = \frac{-64 - 50}{2} = \frac{-114}{2} = -57;$$

$d = a_9 - a_8;$

$$d = -57 - (-64) = -57 + 64 = 7;$$

3) если $a_8 = -7, a_{10} = 3$;

$$a_9 = \frac{a_8 + a_{10}}{2} = \frac{-7 + 3}{2} = \frac{-4}{2} = -2;$$

$$d = a_9 - a_8 = -2 - (-7) = 5;$$

4) если $a_8 = 0,5, a_{10} = -2,5$;

$$a_9 = \frac{a_8 + a_{10}}{2} = \frac{0,5 - 2,5}{2} = -1;$$

$$d = a_9 - a_8 = -1 - 0,5 = -1,5.$$

386.

Запишем данные условия: $a_5 = a_1 + 4d$.

$$\text{Тогда } a_5 = 4,9 + 4 \cdot 9,8 = 44,1 \text{ (м)}.$$

387.

$$\text{Т.к. } a_n = a_1 + (n-1)d,$$

$$\text{то } 105 = 15 + (n-1) \cdot 10;$$

$$90 = 10n - 10;$$

$$10n = 100, \text{ отсюда } n = 10.$$

Ответ: 10 дней.

388.

$$a_n + a_k = a_1 + (n-1)d + a_1 + (k-1)d = 2a_1 + (n+k-2)d,$$

но $a_{n-\ell} + a_{k+\ell} = a_1 + (n-\ell-1)d + a_1 + (k+\ell-1)d = 2a_1 + (n+k-2)d$,

тогда $a_n + a_k = a_{n-1} + a_{k+1}$, доказано,

поэтому $a_{10} + a_5 = a_{10-3} + a_{5-3} = a_7 + a_8 = 30$.

Ответ: $a_{10} + a_5 = 30$.

389.

$$\frac{a_{n+k} + a_{n-k}}{2} = \frac{a_n + a_n}{2} = \frac{2a_n}{2} = a_n \text{ (из предыдущего номера),}$$

$$\text{тогда } a_{20} = \frac{a_{10} + a_{30}}{2} = \frac{120}{2} = 60.$$

390.

1) $a_1 = 1, a_n = 20, n = 50$;

$$S_n = \frac{a_1 + a_n}{2} \cdot n; S_{50} = \frac{1+20}{2} \cdot 50 = (1+20) \cdot 25 = 21 \cdot 25 = 525;$$

2) $a_1 = 1, a_n = 200, n = 100$;

$$S_{100} = \frac{1+200}{2} \cdot 100 = 201 \cdot 50 = 10050;$$

3) $a_1 = -1, a_n = -40, n = 20$;

$$S_{20} = \frac{-1-40}{2} \cdot 20 = -41 \cdot 10 = -410;$$

4) $a_1 = 2, a_n = 100, n = 50$;

$$S_{50} = \frac{2+100}{2} \cdot 50 = 102 \cdot 25 = 2550.$$

391.

$a_n = 98; a_1 = 2; d = 1$. Т.к. $a_n = a_1 + (n - 1)d$, то

$$98 = 2 + (n - 1) \cdot 1;$$

$$96 = n - 1; n = 97;$$

$$S_{97} = \frac{2+98}{2} \cdot 97 = 50 \cdot 97 = 4850.$$

392.

$a_1 = 1; d = 2; a_n = 133$.

Т.к. $a_n = a_1 + (n - 1)d$, то

$$133 = 1 + (n - 1) \cdot 2;$$

$$132 = 2n - 2; n = 67;$$

$$S_{67} = \frac{1+133}{2} \cdot 67 = 67 \cdot 67 = 4489.$$

393.

1) $\underline{a_1 = -5; d = 0,5};$

2) $\underline{a_1 = \frac{1}{2}; d = -3};$

$$S_n = \frac{2a_1 + (n-1)d}{2} \cdot n;$$

$$S_n = \frac{2a_1 + (n-1)d}{2} \cdot n;$$

$$S_{12} = \frac{2 \cdot (-5) + 11 \cdot 0,5}{2} \cdot 12 =$$

$$S_{12} = \frac{2 \cdot \frac{1}{2} + 11 \cdot (-3)}{2} \cdot 12 =$$

$$= (-10 + 5,5) \cdot 6 = -27;$$

$$= (1 - 33) \cdot 6 = -192.$$

394.

1) $a_1 = 9; d = a_2 - a_1 = 13 - 9 = 4;$

$$S_{11} = \frac{2a_1 + 10d}{2} \cdot 11 = \frac{2 \cdot 9 + 10 \cdot 4}{2} \cdot 11 = \frac{(18 + 40) \cdot 11}{2} = 29 \cdot 11 = 319;$$

2) $a_1 = -16; d = a_2 - a_1 = -13 - (-16) = 6$ $S_{12} = \frac{2a_1 + 11d}{2} \cdot 12 =$

$$= \frac{2 \cdot (-16) + 11 \cdot 6}{2} \cdot 12 = (-32 + 66) \cdot 6 = 6 \cdot 34 = 204.$$

395.

1) $a_1 = 3; d = 3; a_n = 273$.

Т.к. $a_n = a_1 + (n - 1)d$, то $273 = 3 + (n - 1) \cdot 3;$

$$270 = 3n - 3; 3n = 273.$$

Тогда $n = 91$.

$$S_{91} = \frac{a_1 + a_{91}}{2} \cdot 91 = \frac{3 + 273}{2} \cdot 91 = 138 \cdot 91 = 12558.$$

$$2) a_1 = 90; d = 80 - 90 = -10; a_n = -60.$$

$$\text{T.k. } a_n = a_1 + (n - 1)d, \text{ то}$$

$$-60 = 90 - 10n + 10;$$

$$10n = 100 + 60 = 160.$$

$$\text{T.e. } n = 16;$$

$$S_{16} = \frac{a_1 + a_{16}}{2} \cdot 16 = (90 - 60) \cdot 8 = 30 \cdot 8 = 240.$$

396.

a) $a_1 = 10; d = 1; a_n = 99.$

Т.к. $a_n = a_1 + (n - 1)d$, то

$99 = 10 + n - 1.$ Тогда $n = 90;$

$$S_{90} = \frac{a_1 + a_{90}}{2} \cdot 90 = \frac{10 + 99}{2} \cdot 90 = 109 \cdot 45 = 4905.$$

б) $a_1 = 100; d = 1; a_n = 999.$

Т.к. $a_n = a_1 + (n - 1)d$, то

$999 = 100 + n - 1.$ Т.е. $n = 900;$

$$S_{900} = \frac{a_1 + a_{900}}{2} \cdot 900 = \frac{100 + 999}{2} \cdot 900 = 1099 \cdot 450 = 494550.$$

397.

1) $a_1 = 3 \cdot 1 + 5 = 8; a_{50} = 3 \cdot 50 + 5 = 155;$

$$S_{50} = \frac{a_1 + a_{50}}{2} \cdot 50 = \frac{8 + 155}{2} \cdot 50 = 163 \cdot 25 = 4075;$$

2) $a_1 = 7 + 2 = 9; a_{50} = 7 + 2 \cdot 50 = 107;$

$$S_{50} = \frac{a_1 + a_{50}}{2} \cdot 50 = \frac{9 + 107}{2} \cdot 50 = 116 \cdot 25 = 2900.$$

398.

$a_1 = 7, a = a_{n+1} - a_n = -3, a_9 = 7 - 3 \cdot 8 = -17.$

Тогда $S_9 = \frac{7 - 17}{2} \cdot 9 = -45.$

399.

$a_1 = 3; d = 1.$

Т.к. $S_n = \frac{2a_1 + (n-1)d}{2} \cdot n$, то $75 = \frac{6 + (n-1)}{2} \cdot n$;

$150 = 6n + n^2 - n;$

$n^2 + 5n - 150 = 0.$ Решим:

$n_1 = 10, n_2 = -15$ – не натуральное.

Ответ: 10.

400.

1) $a_1 = 10; n = 14; S_{14} = 1050.$ 2) $a_1 = 2 \frac{1}{2}; n = 10; S_{10} = 90 \frac{5}{6}.$

Т.к. $S_{14} = \frac{2a_1 + 13d}{2} \cdot 14$, то

Т.к. $S_{10} = \frac{2a_1 + 9 \cdot d}{2} \cdot 10$, то

$$1050 = \frac{20+13d}{2} \cdot 14.$$

Отсюда $1050 = 7(20 + 13d)$;

$$910 = 91d \text{ и } d = 10.$$

Тогда $a_{14} = a_1 + 13d$;

$$a_{14} = 10 + 130 = 140;$$

401.

1) $a_7 = 21$; $S_7 = 205$.

Т.к. $S_7 = \frac{a_1 + a_7}{2} \cdot 7$, то

$$205 = \frac{a_1 + 21}{2} \cdot 7;$$

$$410 = 7a_1 + 147;$$

$$7a_1 = 263.$$

Тогда $a_1 = 37\frac{4}{7}$.

Т.к. $a_7 = a_1 + 6d$, то

$$21 = 37\frac{4}{7} + 6d;$$

$$6d = -16\frac{4}{7};$$

$$d = -\frac{58}{21}.$$

Итак $d = -2\frac{16}{21}$.

402.

$a_n = 12$; $d = 1$; $a_1 = 1$. Т.к. $a_n = a_1 + (n-1)d$, то $12 = 1 + n - 1$.

Тогда $n = 12$. $S_{12} = \frac{a_1 + a_{12}}{2} \cdot 12$; $S_{12} = \frac{1+12}{2} \cdot 12 = 13 \cdot 6 = 78$ (брёвен).

403.

$a_3 + a_9 = a_{1+2} + a_{11-2} = a_1 + a_{11} = 8$ (из предыдущих задач).

$$S_{11} = \frac{a_1 + a_{11}}{2} \cdot 11.$$

$$90\frac{5}{6} = \left(4\frac{2}{3} + 9d\right) \cdot 5.$$

Отсюда $90\frac{5}{6} - 23\frac{1}{3} = 45d$;

$$45d = 67\frac{1}{2} \text{ и } d = 1,5.$$

Тогда $a_{10} = a_1 + 9d$;

$$a_{10} = 2\frac{1}{3} + 13\frac{1}{2} = 15\frac{5}{6}.$$

2) $a_{11} = 92$; $S_{11} = 22$.

Т.к. $S_{11} = \frac{a_1 + a_{11}}{2} \cdot 11$, то

$$22 = \frac{a_1 + 92}{2} \cdot 11;$$

$$44 = (a_1 + 92) \cdot 11;$$

$$a_1 + 92 = 4.$$

Тогда $a_1 = -88$.

Т.к. $a_{11} = a_1 + 10d$, то

$$92 = -88 + 100d;$$

$$180 = 10d.$$

Итак $d = 18$.

Тогда $S_{11} = \frac{8}{2} \cdot 11 = 44$.

404.

Т.к. $S_5 = \frac{2a_1 + 4d}{2} \cdot 5$, т.к. $S_{10} = \frac{2a_1 + 9d}{2} \cdot 10$,

то $65 = \frac{2(a_1 + 2d)}{2} \cdot 5$, то $230 = (2a_1 + 9d) \cdot 5$.

Тогда $13 = a_1 + 2d$. Тогда $2a_1 + 9d = 46 \quad | : 2$,

получим $\begin{cases} a_1 + 2d = 13 \\ 2a_1 + 9d = 46 \end{cases}$; $\begin{cases} 5d = 20 \\ a_1 + 2d = 13 \end{cases}$; $\begin{cases} d = 4 \\ a_1 = 5 \end{cases}$.

405.

$S_{12} = \frac{2a_1 + 11d}{2} \cdot 12$; $S_{12} = 6 \cdot (2a_1 + 11d)$. Тогда

$S_8 - S_4 = \frac{2a_1 + 7d}{2} \cdot 8 - \frac{2a_1 + 3d}{2} \cdot 4 = 4 \cdot (2a_1 + 7d) - 2 \cdot (2a_1 + 3d) =$
 $= 8a_1 + 28d - 4a_1 - 6d = 4a_1 + 22d$;

$3(S_8 - S_4) = 3 \cdot (4a_1 + 22d) = 3 \cdot 2(2a_1 + 11d) = 6 \cdot (2a_1 + 11d)$,

получили: $S_{12} = 3(S_8 - S_4)$.

407.

1) $b_1 = 12, q = 2$;

$b_2 = b_1 \cdot q = 12 \cdot 2 = 24$;

$b_3 = 24 \cdot 2 = 48$;

$b_4 = 48 \cdot 2 = 96$;

$b_5 = 192$;

2) $b_1 = -3, q = -4$;

$b_2 = b_1 \cdot q = -3 \cdot (-4) = 12$;

$b_3 = 12 \cdot (-4) = -48$;

$b_4 = -48 \cdot (-4) = 192$;

$b_5 = 192 \cdot (-4) = -768$.

408.

1) $b_n = 3 \cdot 2^n$.

Пусть $b_{n+1} = 3 \cdot 2^{n+1}$.

Тогда $\frac{b_{n+1}}{b^n} = \frac{3 \cdot 2^{n+1}}{3 \cdot 2^n} = \frac{3 \cdot 2^n \cdot 2}{3 \cdot 2^n} = 2$,

т.к. $\frac{b_{n+1}}{b^n}$ не зависит от n то b_n – геометрическая прогрессия.

2) $b_n = 5^{n+3}$.

Пусть $b_{n+1} = 5^{n+4}$.

Тогда $\frac{b_{n+1}}{b^n} = \frac{5^{n+4}}{5^{n+3}} = \frac{5^n \cdot 5^4}{5^n \cdot 5^3} = 5$ т.к. $\frac{b_{n+1}}{b^n}$ не зависит от n , то

b_n – геометрическая прогрессия.

$$3) \underline{b_n} = \left(\frac{1}{3}\right)^{n-2}.$$

$$\text{Пусть } b_{n+1} = \left(\frac{1}{3}\right)^{n-1};$$

$$\frac{b^{n+1}}{b^n} = \frac{\left(\frac{1}{3}\right)^{n-1}}{\left(\frac{1}{3}\right)^{n-2}} = \frac{\left(\frac{1}{3}\right)^n \cdot \left(\frac{1}{3}\right)^{-1}}{\left(\frac{1}{3}\right)^n \cdot \left(\frac{1}{3}\right)^{-2}} = \frac{1}{\left(\frac{1}{3}\right)^{-1}} = \frac{1}{3} \text{ т.к. } \frac{b_{n+1}}{b_n} \text{ не зависит от } n,$$

то b_n – геометрическая прогрессия.

$$4) \underline{b_n} = \frac{1}{5^{n-1}}.$$

$$\text{Пусть } b_{n+1} = \frac{1}{5^n};$$

$$\frac{b^{n+1}}{b^n} = \frac{\frac{1}{5^n}}{\frac{1}{5^{n-1}}} = \frac{1}{5^n} \cdot 5 = \frac{1}{5},$$

т.к. $\frac{b_{n+1}}{b_n}$ не зависит от n , то b_n – геометрическая прогрессия.

409.

1) Т.к. $b_n = b_1 \cdot q^{n-1}$, то
 $b_4 = b_1 \cdot q^3$, $b_4 = 3 \cdot 10^3 = 3000$.

2) Т.к. $b_n = b_1 \cdot q^{n-1}$, то
 $b_7 = b_1 \cdot q^6 = 4 \cdot \left(\frac{1}{2}\right)^6 = \frac{4}{6} = \frac{1}{16}$.

3) Т.к. $b_n = b_1 \cdot q^{n-1}$, то
 $b_5 = b_1 \cdot q^4 = 1 \cdot (-2)^4 = 16$.

4) Т.к. $b_n = b_1 \cdot q^{n-1}$, то
 $b_6 = b_1 \cdot q^5 = -3 \cdot \left(-\frac{1}{3}\right)^5 = \frac{-3}{-243} = \frac{1}{81}$.

410.

1) $b_1 = 4$; $q = 3$; Т.к. $b_n = b_1 \cdot q^{n-1}$,
то $b_n = 4 \cdot 3^{n-1}$;

$$2) b_1 = 3; q = \frac{1}{3}; \text{Т.к. } b_n = b_1 \cdot q^{n-1}, \text{ то } b_n = 4 \cdot \left(\frac{1}{3}\right)^{n-1};$$

$$3) b_1 = 4; q = -\frac{1}{4}; \text{Т.к. } b_n = b_1 \cdot q^{n-1}, \text{ то } b_n = 4 \cdot \left(-\frac{1}{4}\right)^{n-1};$$

$$4) b_1 = 3; q = -\frac{4}{3}; \text{Т.к. } b_n = b_1 \cdot q^{n-1}, \text{ то } b_n = 3 \cdot \left(-\frac{4}{3}\right)^{n-1}.$$

411.

$$1) \underline{b_1 = 6; b_2 = 12, \dots, b_n = 192};$$

$$q = \frac{b_2}{b_1} = \frac{12}{6} = 2.$$

Т.к. $b_n = b_1 \cdot q^{n-1}$, то
 $192 = 6 \cdot 2^{n-1}$, но $32 = 2^5$, значит,
 $32 = 2^{n-1}$, $2^5 = 2^{n-1}$;

$$5 = n - 1;$$

$$n = 6;$$

$$2) \underline{b_1 = 4; b_2 = 12, \dots, b_n = 324};$$

$$q = \frac{12}{4} = 3.$$

Т.к. $b_n = b_1 \cdot q^{n-1}$, то

$$324 = 4 \cdot 3^{n-1};$$

$$81 = 3^{n-1}, 3^4 = 3^{n-1}, \text{ значит,}$$

$$4 = n - 1;$$

$$n = 5;$$

$$3) \underline{b_1 = 625; b_2 = 125, \dots, b_n = \frac{1}{25}};$$

$$q = \frac{b_2}{b_1} = \frac{125}{625} = \frac{1}{5};$$

Т.к. $b_n = b_1 \cdot q^{n-1}$, то

$$\frac{1}{25} = 625 \cdot \left(\frac{1}{5}\right)^{n-1}, \text{ значит,}$$

$$5^{-2} = 5^4 \cdot 5^{1-n} = 5^{5-n}, \text{ отсюда}$$

$$-2 = 5 - n$$

$$\text{и } n = 7;$$

$$4) \underline{b_1 = -1; b_2 = 2, \dots, b_n = 128};$$

$$q = \frac{b_2}{b_1} = -2 \text{ Т.к. } b_n = b_1 \cdot q^{n-1}, \text{ то}$$

$$128 = -1 \cdot (-2)^{n-1};$$

$$-128 = (-2)^{n-1}, \text{ получили:}$$

$$(-2)^7 = (-2)^{n-1}, \text{ тогда}$$

$$7 = n - 1$$

$$\text{и } n = 8.$$

412.

1) $\underline{b_1 = 2; b_5 = 162.}$

Т.к. $b_5 = b_1 \cdot q^4$, то

$$162 = 2 \cdot q^4;$$

$$81 = q^4;$$

$$3^4 = q^4, \text{ поэтому } q_1 = 3, q_2 = -3; q_1 = \frac{1}{2}, q_2 = -\frac{1}{2};$$

3) $\underline{b_1 = 3; b_4 = 81.}$

Т.к. $b_4 = b_1 \cdot q^3$, то

$$81 = 3 \cdot q^3;$$

$$q^3 = 27 \text{ поэтому } q = 3;$$

2) $\underline{b_1 = -128; b_7 = -2.}$

Т.к. $b_7 = b_1 \cdot q^6$, то

$$-2 = 128 \cdot q^6, \text{ значит } q^6 = \left(\frac{1}{2}\right)^6;$$

$$\frac{1}{64} = q^6;$$

4) $\underline{b_1 = 250; b_4 = -2.}$

Т.к. $b_4 = b_1 \cdot q^3$, то

$$-2 = 250 \cdot q^3;$$

$$q^3 = -\frac{1}{125} \text{ поэтому } q = -\frac{1}{5}.$$

413.

1) $b_1 = 2; q = 3$. Т.к. $b_8 = b_1 \cdot q^7$, то

$$b_8 = 2 \cdot 3^7 = 4374;$$

2) Т.к. $b_n = b_1 \cdot q^{n-1}$ $162 = 2 \cdot 3^{n-1}$;

$$81 = 3^{n-1}, 3^{n-1} = 3^4, \text{ значит,}$$

$$4 = n - 1, n = 5.$$

414.

1) $\underline{b_8 = \frac{1}{9}; b_6 = 81.}$

Т.к. $b_7 = \sqrt{b_8 b_6}$, то

$$b_7 = \sqrt{\frac{1}{9} \cdot 81} = \sqrt{9} = 3.$$

Тогда $q = \frac{b_8}{b_7} = \frac{1}{27}$.

2) $\underline{b_6 = 9; b_8 = 3.}$

Т.к. $b_7 = \sqrt{b_8 b_6}$, то

$$b_7 = \sqrt{9 \cdot 3} = 3\sqrt{3}.$$

Тогда $q = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

415.

1) $\underline{b_4 = 5; b_6 = 20.}$

2) $\underline{b_4 = 9; b_6 = 4.}$

Т.к. $b_5 = \pm\sqrt{b_4 \cdot b_6}$, то

$$b_5 = \pm\sqrt{5 \cdot 20} = \pm 10.$$

Т.к. $b_6 = b_4 \cdot q^2$, то

$$20 = 5 \cdot q^2.$$

Т.к. $b_5 = \pm\sqrt{b_4 \cdot b_6}$, то

$$b_5 = \pm\sqrt{9 \cdot 4} = \pm 6.$$

Т.к. $b_6 = b_4 \cdot q^2$, то

$$q^2 = \frac{20}{5} = 4; 4 = 9 \cdot q^2, q^2 = \frac{9}{4}.$$

Тогда $q^2 = 4$, $q_1 = 2$ или $q_2 = -2$; Тогда $q = \frac{2}{3}$ либо $q = -\frac{2}{3}$;

$$b_4 = b_1 \cdot q^3; 5 = b_1 \cdot (-2)^3.$$

$$b_4 = b_1 \cdot q^3;$$

$$\text{Если } q = 2, \text{ то } b_1 = \frac{5}{8}, b_5 = 10. \quad 9 = b_1 \cdot \left(\frac{2}{3}\right)^3 \text{ либо } 9 = b_1 \cdot \left(\frac{2}{3}\right)^3;$$

$$\text{Если } q = -2, \text{ то } b_1 = -\frac{5}{8}.$$

$$b_1 = 9 \cdot \frac{27}{8} = \frac{243}{8} = 30\frac{3}{8} \text{ либо}$$

$$b_5 = -10, b_1 = -\frac{5}{8}.$$

$$b_1 = -30\frac{3}{8}.$$

$$\text{Ответ: } b_5 = 10, b_1 = \frac{5}{8},$$

$$\text{Ответ: } b_5 = 6, b_1 = 30\frac{3}{8};$$

$$b_5 = -10, b_1 = -\frac{5}{8}.$$

$$b_5 = -6, b_1 = -30\frac{3}{8}.$$

416.

$$q = 1,2$$

$$b_2 = 300000 \cdot 1,2 = 360000.$$

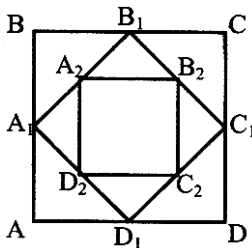
$$\text{Тогда } 300000 + 360000 = 660000 \text{ р.}$$

$$660000 \cdot 1,2 = 792000.$$

$$\text{Отсюда } 660000 + 792000 = 1452000 \text{ р.}$$

Ответ: 1 452 000 р.

417.



ABCD – квадрат,

AB = 4 см,

A_1, B_1, C_1, D_1 – середины соответствующих сторон.
Докажем, что $S_A, S_{A_1}, S_{A_2}, \dots$ – геометрическая прогрессия.

и найдем S_7

$$AB = 4 \text{ см}, A_1B_1 = 2\sqrt{2} \text{ см}, A_2B_2 = 2 \text{ см}, A_3B_3 = \sqrt{2} \text{ см}.$$

$$\left. \begin{aligned} \frac{A_1B_1}{AB} &= \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} \\ \frac{A_2B_2}{A_1B_1} &= \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{aligned} \right\} \text{значит,}$$

$$b_1 = 4; \quad q = \frac{\sqrt{2}}{2};$$

$$S_1 = b_1^2; \quad S_n = b_n^2;$$

$$b_n = 4 \cdot \left(\frac{\sqrt{2}}{2}\right)^{n-1}. \quad \text{Т.к.} \quad \begin{aligned} S_7 &= (b_7)^2, \text{ то} \\ S_7 &= (b_7)^2 = (b_1 \cdot q^6)^2 = b_1^2 \cdot q^{12}; \end{aligned}$$

$$b_n = 8 \left(\sqrt{2}\right)^{n-1}. \quad \text{Тогда } S_7 = 4^2 \cdot \left(\frac{\sqrt{2}}{2}\right)^{12} = 2^4 \cdot 2^{-6} = 2^{-2} = \frac{1}{4} \text{ см}^2.$$

$$\text{Ответ: } 8 \left(\sqrt{2}\right)^{n-1}; \quad S_7 = \frac{1}{4} \text{ (см}^2\text{)}.$$

419.

1) Если b_1, b_2, b_3 – члены геометрической прогрессии, то $b_2^2 = b_1 \cdot b_3$, т.е. $\cos^2 \alpha = (1 - \sin \alpha)(1 + \sin \alpha) = 1 - \sin^2 \alpha = \cos^2 \alpha$, это верно.

2) Докажем, что $\frac{b_2}{b_1} = \frac{b_3}{b_2}$;

$$\frac{\cos \alpha}{1 - \sin \alpha} = \frac{\cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}}{\left(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}\right)^2} = \frac{\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}},$$

$$\text{Но и } \frac{1 + \sin \alpha}{\cos \alpha} = \frac{\left(\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}\right)^2}{\cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}} = \frac{\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}},$$

получили, что

b_1, b_2, b_3 – геометрическая прогрессия.

420.

$$1) \underline{b_1 = \frac{1}{2}; q = 2; n = 6.}$$

$$\text{Т.к. } S_n = \frac{b_1(1-q^n)}{1-q}, \text{ то}$$

$$S_6 = \frac{\frac{1}{2}(1-2^6)}{1-2} = \frac{1-64}{-2} = 31,5;$$

$$2) \underline{b_1 = -2; q = \frac{1}{2}; n = 5.}$$

$$\text{Т.к. } S_5 = \frac{b_1(1-q^5)}{1-q}, \text{ то}$$

$$S_5 = \frac{-2 \cdot \left(1 - \frac{1}{32}\right)}{1 - \frac{1}{2}} = -4 \cdot \frac{31}{32} = -\frac{31}{8};$$

$$3) \underline{b_1 = 1; q = -\frac{1}{3}; n = 4.}$$

$$\text{Т.к. } S_4 = \frac{b_1(1-q^4)}{1-q}, \text{ то}$$

$$S_4 = \frac{1 \cdot \left(1 - \left(-\frac{1}{3}\right)^4\right)}{1 + \frac{1}{3}} =$$

$$= \frac{80 \cdot 3}{81 \cdot 4} = \frac{20}{27}$$

$$4) \underline{b_1 = -5; q = -\frac{2}{3}; n = 5.}$$

$$\text{Т.к. } S_5 = \frac{b_1(1-q^5)}{1-q}, \text{ то}$$

$$S_5 = \frac{-5 \cdot \left(1 - \left(-\frac{2}{3}\right)^5\right)}{1 + \frac{2}{3}} =$$

$$= \frac{-5 \cdot \left(1 + \frac{32}{243}\right)}{\frac{5}{3}} = -\frac{275}{81}$$

$$5) \underline{b_1 = 6; q = 1; n = 200.}$$

т.к. $q = 1$, то прогрессия вырождена и $S_{200} = 6 \cdot 200 = 1200$.

$$6) \underline{b_1 = -4; q = 1; n = 100.}$$

т.к. $q = 1$, то прогрессия вырождена и $S_{200} = -4 \cdot 100 = -400$.

421.

$$1) b_1 = 5; q = 2. \text{ Т.к. } S_7 = \frac{b_1(1-q^7)}{1-q}, \text{ то}$$

$$S_7 = \frac{5 \cdot (1-2^7)}{1-2} = -5(1-128) = 635;$$

$$2) b_1 = 2; q = 3. \text{ Т.к. } S_7 = \frac{b_1(1-q^7)}{1-q}, \text{ то}$$

$$S_7 = \frac{2 \cdot (1-3^7)}{1-3} = 3^7 - 1 = 2187 - 1 = 2186;$$

422.

1) Т.к. $b_7 = b_1 \cdot q^6$ и $q = 2$, то Т.к. $S_7 = \frac{b_1(1-q^7)}{1-q}$, то

$$b_7 = 5 \cdot 64; -635 = b_1(1 - 128).$$

Тогда $b_7 = 320$; $b_1 = -635 : (-127) = 5$.

Ответ: $b_7 = 320$, $b_1 = 5$.

2)

а) Т.к. $\frac{b_1(1-q^8)}{1-q} = S_8$, то

$$85 \cdot 3 = b_1 \cdot (1 - 256).$$

Тогда $b_1 = (85 \cdot 3) / (-255) = 255 / (-255) = -1$.

б) Т.к. $b_8 = b_1 \cdot q^7$, то

$$b_8 = (-1) \cdot (-2)^7 = 128.$$

Ответ: $b_1 = -1$, $b_8 = 128$.

423.

1) $S_n = 189$, $b_1 = 3$, $q = 2$.

2) $S_n = 635$, $b_1 = 5$, $q = 2$.

Т.к. $S_n = \frac{b_1(1-q^n)}{1-q}$, то

Т.к. $635 = \frac{5 \cdot (1-2^n)}{1-2}$, то

$$189 = \frac{3 \cdot (1-2^n)}{1-2};$$

$$-635 = 5 \cdot (1-2^n);$$

$$-189 = 3 \cdot (1-2^n);$$

$$-127 = 1-2^n;$$

$$-63 = 1-2^n;$$

$$-128 = 2^n;$$

$$-64 = -2^n;$$

$$2^7 = 2^n, \text{ ПОЭТОМУ}$$

$$2^n = 2^6, \text{ ПОЭТОМУ}$$

$$n = 7;$$

$$n = 6;$$

3) $S_n = 170$, $b_1 = 256$, $q = -\frac{1}{2}$.

Т.к. $S_n = \frac{b_1(1-q^n)}{1-q}$, то $170 = \frac{256 \cdot \left(1 - \left(-\frac{1}{2}\right)^n\right)}{\frac{3}{2}}$;

$$510 = 512 \cdot \left(1 - \left(-\frac{1}{2}\right)^n\right), \text{ тогда } 510 = 512 - 512 \left(-\frac{1}{2}\right)^n;$$

$$512 \left(-\frac{1}{2}\right)^n = 2; \left(-\frac{1}{2}\right)^n = \frac{1}{256}; \left(-\frac{1}{2}\right)^n = \left(-\frac{1}{2}\right)^8; n = 8;$$

$$4) \underline{S_n = -99, b_1 = -9, q = -2. \text{ Т.к. } S_n = \frac{b_1(1-q^n)}{1-q}, \text{ то}}$$

$$-99 = \frac{-9 \cdot (1 - (-2)^n)}{1 - (-2)}; \quad 33 = 1 - (-2)^n;$$

$$-99 = \frac{-9 \cdot (1 - (-2)^n)}{3}; \quad (-2)^5 = (-2)^n;$$

$$n = 5.$$

424.

$$1) \underline{b_1 = 7, q = 3, S_n = 847.}$$

$$\text{Т.к. } S_n = \frac{b_1(1-q^n)}{1-q}, \text{ то}$$

$$847 = 7 \cdot 121 = \frac{7 \cdot (1 - 3^n)}{-2};$$

$$121 \cdot (-2) = 1 - 3^n;$$

$$243 = 3^n;$$

$$3^5 = 3^n, \text{ поэтому}$$

$$n = 5; b_5 = 7 \cdot 3^4 = 567;$$

$$2) \underline{b_1 = 8, q = 2, S_n = 4088.}$$

$$\text{Т.к. } S_n = \frac{b_1(1-q^n)}{1-q}, \text{ то } 4088 = 8 \cdot 511 = \frac{8 \cdot (1 - 2^n)}{1 - 2};$$

$$-511 = 1 - 2^n, 512 = 2^n;$$

$$\text{поэтому } 2^9 = 2^n;$$

$$n = 9; b_9 = 8 \cdot 2^8 = 2048;$$

3)

$$\underline{b_1 = 2, b_n = 1458, S_n = 2186.}$$

$$\text{Т.к. } b_n = b_1 \cdot q^{n-1}, \text{ то}$$

$$\text{Т.к. } S_n = \frac{b_1(1-q^n)}{1-q}, \text{ то}$$

$$1458 = 2 \cdot q^{n-1};$$

$$2186 = \frac{2 \cdot (1 - q^n)}{1 - q};$$

$$729 = q^{n-1},$$

$$\text{получим } q^n = 729q;$$

$$1093(1 - q) = 1 - q^n;$$

$$1093 - 1093q - 1 + q^n = 0,$$

$$\text{т.к. } q^n = 729q, \text{ то}$$

$$1092 - 1093q + 729q = 0;$$

$$1092 - 364q = 0;$$

$$q = 3, \text{ тогда}$$

$$3^{n-1} = 3^6, n = 7;$$

4) $b_1 = 1, b_n = 2401, S_n = 2801.$

Т.к. $b_n = b_1 \cdot q^{n-1}$, то

$2401 = q^{n-1};$

$q^n = 2401q.$

Т.к. $S_n = \frac{b_1(1-q^n)}{1-q}$, то

$2801 = \frac{1-q^n}{1-q};$

$2801(1-q) = 1 - q^n,$

т.к. $q^n = 2401q$, то

$2801(1-q) = 1 - 2401q;$

$2800 = 2801q - 2401q;$

$2800 = 400q;$

$q = 7; q^{n-1} = 2401$, тогда

$7^{n-1} = 7^4$, значит, $n = 5.$

425.

1) $b_1 = 1; q = 2; b_n = 128.$

Т.к. $b_n = b_1 \cdot q^{n-1}$, то

$128 = 2^{n-1}, 2^7 = 2^{n-1}$, значит,

$n = 8.$

Т.к. $S_8 = \frac{b_1(1-q^8)}{1-q}$, то

$S_8 = \frac{1 \cdot (1-2^8)}{1-2} = -(1-256) = 255;$

2) $b_1 = 1; b_2 = 3; q = 3; b_n = 243.$

Т.к. $b_n = b_1 \cdot q^{n-1}$, то

$243 = 1 \cdot 3^{n-1}, 3^5 = 3^{n-1}$, тогда

$n = 6.$

Т.к. $S_6 = \frac{b_1(1-q^6)}{1-q}$, то

$S_6 = \frac{1 \cdot (1-3^6)}{1-3} = \frac{728}{2} = 364;$

3) $b_1 = -1; q = -2; b_n = 128.$

Т.к. $b_n = b_1 \cdot q^{n-1}$,

то $128 = -1 \cdot (-2)^{n-1};$

$-128 = (-2)^{n-1};$

$(-2)^7 = (-2)^{n-1}$, значит $n = 8.$

Т.к. $S_8 = \frac{b_1(1-q^8)}{1-q}$, то $S_8 = \frac{1 \cdot (1-256)}{3} = \frac{255}{3} = 85.$

$$4) b_1 = 5; q = -3; b_n = 405.$$

$$\text{Т.к. } b_n = b_1 \cdot q^{n-1}, \text{ то}$$

$$405 = 5 \cdot (-3)^{n-1}, (-3)^{n-1} = 81 = 3^4, n = 5.$$

$$\text{Т.к. } S_5 = \frac{b_1(1-q^5)}{1-q}, \text{ то } S_5 = \frac{5 \cdot (1+243)}{4} = 5 \cdot 61 = 305.$$

426.

$$1) \text{ Т.к. } b_3 : b_2 = q, \text{ то } q = \frac{25}{15} = \frac{5}{3}.$$

$$\text{Т.к. } b_5 = b_2 \cdot q^3, \text{ то } b_5 = 15 \cdot \frac{125}{27} = \frac{625}{9}.$$

$$\text{Т.к. } b_1 = b_2 : q, \text{ то } b_1 = 15 : \frac{5}{3} = 9.$$

$$S_4 = \frac{b_1(1-q^4)}{1-q} = \frac{9 \cdot \left(1 - \frac{625}{81}\right)}{1 - \frac{5}{3}} = -\frac{544}{9} : \left(-\frac{2}{3}\right) = \frac{544 \cdot 3}{9 \cdot 2} = \frac{272}{3} = 90\frac{2}{3}.$$

$$2) \text{ Т.к. } b_4 = b_2 \cdot q^2, \text{ то } b_1 = b_2 : q,$$

$$686 : 14 = q^2; b_1 = 14 : 7 = 2;$$

$$q^2 = 49 \Rightarrow q = 7, \text{ т.к. } q > 0;$$

$$b_5 = b_4 \cdot q.$$

$$\text{Тогда } S_4 = \frac{2 \cdot (1-7^4)}{1-7} = \frac{2(1-7^4)}{-6} = \frac{1-7^4}{-3} = 800;$$

$$b_5 = 686 \cdot 7 = 4802.$$

427.

$$1) b_1 = 3; q = 2. \text{ Т.к. } S_5 = \frac{b_1(1-q^5)}{1-q}, \text{ то}$$

$$S_5 = \frac{3 \cdot (1-32)}{1-2} = -3(1-32) = -3 \cdot (-31) = 93;$$

$$2) b_1 = 3; b_2 = -\frac{1}{2}.$$

$$\text{Т.к. } b_2 : b_1 = q, \text{ то } q = \frac{1}{2}. \text{ Т.к. } S_6 = \frac{b_1(1-q^6)}{1-q}, \text{ то}$$

$$S_6 = \frac{-1 \cdot \left(1 - \frac{1}{64}\right)}{1 - \frac{1}{2}} = -2 \cdot \left(1 - \frac{1}{64}\right) = -2 \cdot \frac{63}{64} = -1\frac{31}{32}.$$

428.

$$(x-1)(x^{n-1} + x^{n-2} + x^{n-3} + \dots + 1) = x^n + x^{n-1} + x^{n-2} + \dots + x - x^{n-1} - x^{n-2} - \dots - x - 1 = x^n - 1.$$

429.

$$1) \begin{cases} b_3 = b_1 q^2 \\ S_3 = \frac{b_1(1-q^3)}{1-q} \end{cases} \Rightarrow \begin{cases} 135 = b_1 q^2 \\ 195 = \frac{b_1(1-q^3)}{1-q} \end{cases} \Rightarrow \begin{cases} 135 = b_1 q^2 \\ 195 = b_1(1+q+q^2) \end{cases}.$$

Поделим 1 на 2 уравнение

$$\frac{135}{195} = \frac{q^2}{1+q+q^2}, \text{ тогда}$$

$$\frac{9}{13} = \frac{q^2}{1+q+q^2};$$

$$13q^2 - 9q^2 - 9q - 9 = 0;$$

$$4q^2 - 9q - 9 = 0.$$

Решим:

$$q = \frac{9 \pm \sqrt{81 + 4 \cdot 4 \cdot 9}}{8} = \frac{9 \pm 15}{8}, \text{ т.е.}$$

$$q = 3 \text{ или } q = -\frac{3}{4}. \text{ Если } q = 3, \text{ то}$$

$$b_1 = \frac{135}{9} = 15, \text{ и } b_1 = \frac{135}{\left(\frac{3}{4}\right)^2} = \frac{135 \cdot 16}{9} = 240, \text{ если } q = -\frac{3}{4}.$$

$$\text{Ответ: } q = 3, b_1 = 15 \text{ или } q = -\frac{3}{4}, b_1 = 240.$$

$$2) \text{ Т.к. } S_3 = \frac{b_1 \cdot (1-q^3)}{1-q}, \text{ то}$$

$$372 = \frac{12 \cdot (1-q^3)}{1-q}, q \neq 1;$$

$$1 + q + q^2 = 31;$$

$$q^2 + q - 30 = 0.$$

Решим:

$$q = -6, q_2 = 5. \text{ Если } q_1 = -6, \text{ то}$$

$$b_3 = 12 \cdot (-6)^2 = 432, \text{ и } b_3 = 12 \cdot 5^2 = 300, \text{ если } q_2 = 5.$$

$$\text{Ответ: } q = -6, b_3 = 432 \text{ или } q = 5, b_3 = 300.$$

430.

1) Т.к. $b_3 = b_1 \cdot q^2$, $b_5 = b_1 \cdot q^4$ и

$b_3 + b_5 = 90$, то

$b_1 \cdot q^2 + b_1 \cdot q^4 = 90$, тогда

$$q^2 + q^4 - 90 = 0.$$

Обозначим $q^2 = t$, получим $t^2 + t - 90 = 0$. Решим:

$$t_1 = 9; t_2 = -10.$$

Тогда $q^2 = 9$ т.к. $q^2 = -10$ не имеет решения.

Поэтому $q_1 = 3$; $q_2 = -3$.

Ответ: $q = 3$ или $q = -3$.

2) Т.к. $b_4 = b_2 \cdot q^2$, $b_6 = b_2 \cdot q^4$ и $b_4 + b_6 = 60$, то

$b_2 \cdot q^2 + b_2 \cdot q^4 = 60$, тогда

$$3q^2 + 3q^4 - 60 = 0;$$

$$q^4 + q^2 - 20 = 0.$$

Обозначим $q^2 = t$, значит $t^2 + t - 20 = 0$. Решим:

$$t_1 = 4; t_2 = -5.$$

Тогда $q^2 = 4$ т.к. $q^2 = -5$ не имеет решения.

Поэтому $q_1 = 2$; $q_2 = -2$.

Ответ: $q = 2$ или $q = -2$.

$$3) \begin{cases} b_1 - b_3 = 15 \\ b_2 - b_4 = 30 \end{cases} \begin{cases} b_1 - b_1 q^2 = 15 \\ b_1 q - b_1 q^3 = 30 \end{cases} \begin{cases} b_1 \cdot (1 - q^2) = 15 \\ b_1 \cdot q(1 - q^2) = 30 \end{cases} \begin{cases} \frac{b_1}{b_1 q} = \frac{15}{30} \\ b_1 \cdot (1 - q^2) = 15 \end{cases}$$

$$\begin{cases} \frac{1}{q} = \frac{1}{2} \\ b_1 \cdot (1 - q^2) = 15 \end{cases} \begin{cases} q = 2 \\ b_1 = -5 \end{cases}$$

$$\text{Значит, } S_{10} = \frac{b_1 \cdot (1 - q^{10})}{1 - q} = \frac{-5 \cdot (1 - 2^{10})}{1 - 2} = 5 \cdot (1 - 1024) = -5115.$$

$$4) \begin{cases} b_3 - b_1 = 24 \\ b_5 - b_1 = 624 \end{cases} \begin{cases} b_1 \cdot q^2 - b_1 = 24 \\ b_1 \cdot q^4 - b_1 = 624 \end{cases} \begin{cases} b_1 \cdot (q^2 - 1) = 24 \\ b_1 \cdot (q^4 - 1) = 624 \end{cases}$$

Поделим 1 на 2 уравнение

$$\frac{q^2 - 1}{q^4 - 1} = \frac{24}{624}.$$

$$\text{Тогда } \frac{q^2 - 1}{(q^2 + 1)(q^2 - 1)} = \frac{1}{26};$$

$$q^2 + 1 = 26;$$

$$q^2 = 25, q_1 = 5; q_2 = -5, b_1 = \frac{24}{24} = 1.$$

$$\text{Если } q = 5, \text{ то } S_5 = \frac{b_1 \cdot (1 - q^5)}{1 - q} = \frac{1 - 5^5}{1 - 5} = \frac{1 - 3125}{-4} = 781.$$

$$\text{Если } q = -5, \text{ то } S_5 = \frac{b_1 \cdot (1 - q^5)}{1 - q} = \frac{1(1 + 3125)}{6} = \frac{3126}{6} = 521.$$

Ответ: $S_5 = 781$, если $q = 5$; $S_5 = 521$, если $q = -5$.

431.

$$1) \underline{b_1 = 1; b_2 = \frac{1}{2}; b_3 = \frac{1}{4}; \dots}$$

$$q = \frac{b_2}{b_1} = \frac{1/2}{1} = \frac{1}{2} < 1, \text{ значит прогрессия бесконечно убывает;}$$

$$2) \underline{b_1 = \frac{1}{3}; b_2 = \frac{1}{9}; b_3 = \frac{1}{27} \dots}$$

$$q = \frac{b_2}{b_1} = \frac{1/9}{1/3} = \frac{1}{3} < 1, \text{ значит, прогрессия бесконечно убывает;}$$

$$3) \underline{b_1 = -81; b_2 = -27; \dots}$$

$$q = \frac{b_2}{b_1} = \frac{-27}{-81} = \frac{1}{3} < 1, \text{ значит, прогрессия бесконечно убывает;}$$

$$4) \underline{b_1 = -16; b_2 = -8; \dots}$$

$$q = \frac{b_2}{b_1} = \frac{-8}{-16} = \frac{1}{2} < 1, \text{ значит, прогрессия бесконечно убывает.}$$

432.

$$1) \underline{b_1 = 40; b_2 = 20; \dots}$$

$$q = \frac{b_2}{b_1} = \frac{-20}{40} = -\frac{1}{2}; |q| = \frac{1}{2} < 1, \text{ значит, прогрессия бесконечно}$$

убывает;

$$2) \underline{b_7 = 12; b_{11} = \frac{3}{4}; \dots}$$

$$\text{Т.к. } b_{11} = b_7 \cdot q^4, \text{ то } \frac{3}{4} = 12 \cdot q^4.$$

$$\text{Тогда } q^4 = \frac{1}{16} \text{ и } q = \frac{1}{2} \text{ или } q = -\frac{1}{2}, |q| = \frac{1}{2} < 1, \text{ значит, прогрес-}$$

сия бесконечно убывает;

$$3) \underline{b_7 = -30; b_6 = 15; \dots}$$

$$q = \frac{b_2}{b_1} = \frac{-30}{15} = -2; |q| = 2 > 1, \text{ значит, прогрессия не бесконечно}$$

убывающая;

$$4) \underline{b_5 = -9; b_2 = -\frac{1}{27}; \dots}$$

$$\text{Т.к. } b_5 = b_2 \cdot q^3, \text{ то } -\frac{1}{27} = -9 \cdot q^3.$$

$$\text{Отсюда } q^3 = \frac{1}{243}.$$

$$\text{Тогда } q = \pm \frac{1}{3\sqrt[3]{3}}, |q| = \frac{1}{3\sqrt[3]{3}} < 1, \text{ значит, прогрессия бесконечно}$$

убывает.

433.

$$1) 1; \frac{1}{3}; \frac{1}{9} \dots$$

$$q = \frac{1}{3}, \text{ т.к. } |q| < 1, \text{ то } S = \frac{b_1}{1-q}; S = \frac{1}{1-\frac{1}{3}} = \frac{3}{2};$$

$$2) 6; 1; \frac{1}{6} \dots$$

$$q = \frac{1}{6}, \text{ т.к. } |q| < 1, \text{ то } S = \frac{b_1}{1-q}, \text{ т.е. } S = \frac{6}{1-\frac{1}{6}} = \frac{6 \cdot 6}{5} = \frac{36}{5};$$

$$3) -25; -5; -1 \dots$$

$$q = \frac{1}{5}, \text{ т.к. } |q| < 1, \text{ то } S = \frac{b_1}{1-q}, \text{ т.е.}$$

$$S = \frac{-25}{1-\frac{1}{5}} = \frac{-25 \cdot 5}{4} = -\frac{125}{4};$$

$$4) -7; -1; -\frac{1}{7} \dots$$

$$q = \frac{1}{7}, \text{ т.к. } |q| < 1, \text{ то } S = \frac{b_1}{1-q}, \text{ т.е.}$$

$$S = \frac{-7}{1-\frac{1}{7}} = \frac{-7 \cdot 7}{6} = -\frac{49}{6}.$$

434.

$$1) b_1 = \frac{1}{8}, q = \frac{1}{2}, S = \frac{b_1}{1-q}; \quad 2) b_1 = 9, q = -\frac{1}{3}, S = \frac{b_1}{1-q};$$

$$S = \frac{\frac{1}{8}}{1-\frac{1}{2}} = \frac{2}{8} = \frac{1}{4};$$

$$S = \frac{9}{1+\frac{1}{3}} = \frac{9 \cdot 3}{4} = 6\frac{3}{4};$$

$$3) b_5 = \frac{1}{81}, q = \frac{1}{3}.$$

$$4) b_4 = -\frac{1}{8}, q = -\frac{1}{2}.$$

Т.к. $b_5 = b_1 \cdot q^4$, то

Т.к. $b_4 = b_1 \cdot q^3$, то

$$\frac{1}{81} = b_1 \cdot \left(\frac{1}{3}\right)^4, b_1 = 1.$$

$$-\frac{1}{8} = b_1 \cdot \left(-\frac{1}{2}\right)^3, b_1 = 1.$$

$$\text{Тогда } S = \frac{1}{1-\frac{1}{3}} = \frac{3}{2} = 1,5;$$

$$\text{Тогда } S = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}.$$

435.

$$1) \underline{b_n} = 3 \cdot (-2)^n;$$

$$b_1 = -6; b_2 = 12;$$

$$q = \frac{b_2}{b_1} = \frac{12}{-6} = -2. \text{ Т.к. } |q| = 2 > 1, \text{ то } b_n \text{ — не бесконечно убывающая геометрическая прогрессия;}$$

$$2) \underline{b_n} = -3 \cdot 4^n; b_1 = -12; b_2 = -48;$$

$$q = \frac{b_2}{b_1} = \frac{-48}{-12} = 4. \text{ Т.к. } |q| = 4 > 1, \text{ то } b_n \text{ — не бесконечно убывающая геометрическая прогрессия;}$$

$$3) \underline{b_n} = 2 \cdot \left(-\frac{1}{3}\right)^{n-1};$$

$$b_1 = 2; b_2 = -\frac{2}{3}; q = \frac{b_2}{b_1} = \frac{-\frac{2}{3}}{2} = -\frac{1}{3}.$$

Т.к. $|q| = \frac{1}{3} < 1$, то b_n — бесконечно убывающая геометрическая прогрессия;

$$4) \underline{b_n} = 5 \cdot \left(-\frac{1}{2}\right)^{n-1};$$

$b_1 = 5; q = -\frac{1}{2}$. Т.к. $|q| = \frac{1}{2} < 1$, то b_n — бесконечно убывающая геометрическая прогрессия.

436.

1) $b_1 = 12, q = \frac{1}{3};$

$$S = \frac{b_1}{1-q}; S = \frac{12}{1-\frac{1}{3}} = \frac{12 \cdot 3}{2} = 18;$$

2) $b_1 = 100, q = \frac{1}{10};$

$$S = \frac{b_1}{1-q}; S = \frac{100}{1+\frac{1}{10}} = \frac{1000}{11} = 90\frac{10}{11}.$$

437.

1) Т.к. $b_5 = b_1 \cdot q^4$, то $\frac{\sqrt{2}}{16} = b_1 \cdot \frac{1}{2^4} = \frac{b_1}{16}$, значит,

$$b_1 = \sqrt{2}. \text{ Тогда } S = \frac{\sqrt{2}}{1-\frac{1}{2}} = 2\sqrt{2}.$$

Ответ: $S = 2\sqrt{2}$.

2) Т.к. $b_4 = b_1 \cdot q^3$, то $\frac{9}{8} = b_1 \cdot \left(\frac{\sqrt{3}}{2}\right)^3$,

отсюда $b_1 = \frac{9}{8} \cdot \frac{8}{3\sqrt{3}} = \sqrt{3};$

$$\text{и } S = \frac{\sqrt{3}}{1-\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{2-\sqrt{3}} = \frac{2\sqrt{3}(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{4\sqrt{3}+6}{1} = 4\sqrt{3}+6.$$

Ответ: $S = 4\sqrt{3} + 6$.

438.

а) Т.к. $S = \frac{b_1}{1-q}$, то $150 = \frac{b_1}{1-\frac{1}{3}}$, $150 \cdot \frac{2}{3} = b_1;$

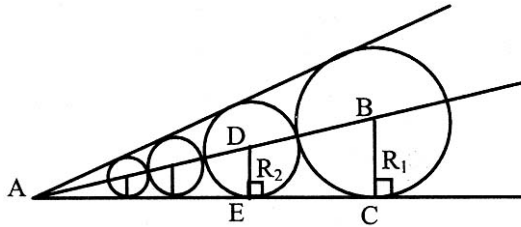
$b_1 = 100;$

б) Т.к. $S = \frac{b_1}{1-q}$, то $150 = \frac{75}{1-q}$. Тогда $2 = \frac{1}{1-q}; 1-q = \frac{1}{2}, q = \frac{1}{2}.$

439.

Т.к. $b_1 = a; q = \frac{1}{2}$, то получим $S = \frac{a}{1-\frac{1}{2}} = 2a.$

440.



Так как все окружности вписаны в угол, то их центры лежат на биссектрисе угла A .

$$\text{Тогда } AB = \frac{R}{\sin 30^\circ} = 2R = \frac{R}{1/2} = 2R, AD = AB + R_1 + R_2.$$

$$\text{Т.к. } \triangle ADE \sim \triangle ABC, \text{ то получаем } \frac{AB}{AD} = \frac{R_1}{R_2}.$$

$$\text{Тогда } \frac{2R_1}{R_1 - R_2} = \frac{R_1}{R_2}, \quad R_2 = \frac{1}{3}R_1.$$

Действуя аналогично, рассматривая подобные треугольники, получим что $R_n = R_1 \cdot \left(\frac{1}{3}\right)^{n-1}$.

Покажем, что $R_1 + 2(R_2 + R_3 + \dots + R_n + \dots) = 2R_1$. Пусть $b_n = R_{n+1}$, $q = 1/3$.

$$\text{Тогда } S = \frac{b_1}{1-q}.$$

$$\text{Значит } S = \frac{R_2}{1 - 1/3} = \frac{3R_2}{2} = \frac{3 \cdot \frac{1}{3}R_1}{2} = \frac{R_1}{2},$$

$$\text{отсюда } R_1 + 2S = R_1 + 2 \cdot \frac{R_1}{2} = 2R_1.$$

441.

$$1) 0,(5) = 0,5555\dots$$

Обозначим через

$$y = 0,(5). \text{ Умножим обе части равенства на } 10.$$

$$\text{Тогда } 5 + 0,(5) = 10y, \text{ но } y = 0,(5), \text{ следовательно,}$$

$$5 + y = 10y,$$

$$5 = 9y.$$

$$\text{Итак, } y = \frac{5}{9}, \Rightarrow 0,(5) = \frac{5}{9}.$$

$$2) 0,(9) = 0,999\dots$$

Обозначим через $y = 0,(9)$,

умножим на 10,

$$9 + 0,(9) = 10y;$$

$$9 + y = 10y;$$

$$9 = 9y, \text{ тогда } y = 1; 0,(9) = 1;$$

$$3) 0,(12) = 0,1212\dots$$

Обозначим через

$y = 0,(12)$, умножим на 100:

$$12 + 0,(12) = 100y, 0,(12) = y, \text{ значит,}$$

$$12 + y = 100y;$$

$$12 = 99y;$$

$$y = \frac{12}{99} = \frac{4}{33}.$$

$$\text{Отсюда } 0,(12) = \frac{4}{33}.$$

$$4) 0,2(3) = 0,2333\dots$$

Обозначим через

$$0,(3) = y, 0,2(3) = 0,2 + 0,0(3) = 0,2 + 0,(3) \cdot 0,1.$$

Вычислим $0,(3)$, затем искомое

$$3 + 0,(3) = 10y;$$

$$3 = 9y;$$

$$y = \frac{1}{3}. \text{ Тогда } 0,2(3) = 0,2 + \frac{1}{3} \cdot 0,1 = \frac{1}{5} + \frac{1}{30} = \frac{7}{30}.$$

446.

$$1) \underline{a_n = n(n+3)};$$

$$n = 1, a_1 = 1 \cdot (1 + 3) = 4; n = 2, a_2 = 2 \cdot (2 + 3) = 2 \cdot 5 = 10;$$

$$n = 3, a_3 = 3 \cdot (3 + 3) = 3 \cdot 6 = 18;$$

$$2) \underline{a_n = 4^n};$$

$$n = 1, a_1 = 4; n = 2, a_2 = 16; n = 3, a_3 = 64;$$

$$3) \underline{a_n = 5 \cdot 2^n};$$

$$n = 1, a_1 = 5 \cdot 2 = 10; n = 2, a_2 = 5 \cdot 2^2 = 5 \cdot 4 = 20;$$

$$n = 3, a_3 = 5 \cdot 2^3 = 5 \cdot 8 = 40;$$

$$4) \underline{a_n = \sin \frac{\pi}{n}};$$

$$n = 1, a_1 = \sin \pi = 0; n = 2, a_2 = \sin \frac{\pi}{2} = 1;$$

$$n = 3, a_3 = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

447.

$$1) a_n = \frac{n-1}{n+1};$$

$$n = 10, a_{10} = \frac{10-1}{10+1} = \frac{9}{11}; n = 30,$$

$$a_{30} = \frac{30-1}{30+1} = \frac{29}{31};$$

$$2) a_n = \frac{n+9}{2n-1};$$

$$n = 10, a_{10} = \frac{10+9}{2 \cdot 10 - 1} = \frac{19}{19} = 1; n = 30, a_{30} = \frac{30+9}{2 \cdot 30 - 1} = \frac{39}{59};$$

$$3) a_n = |n - 15| - 5;$$

$$n = 10, a_{10} = |10 - 15| - 5 = 0; n = 30,$$

$$a_{30} = |30 - 15| - 5 = 10;$$

$$4) a_n = 10 - |n - 20|;$$

$$n = 10, a_{10} = 10 - |10 - 20| = 0; n = 30, a_{30} = 10 - |30 - 20| = 0.$$

448.

$$a_2 = 1 - 0,5 \cdot a_1 = 1 - 0,5 \cdot 2 = 0;$$

$$a_4 = 1 - 0,5 \cdot a_3 = \frac{1}{2};$$

$$a_6 = 1 - 0,5 \cdot a_5 = 1 - 0,5 \cdot \frac{3}{4} = 1 - \frac{3}{8} = \frac{5}{8};$$

$$a_3 = 1 - 0,5 \cdot a_2 = 1;$$

$$a_5 = 1 - 0,5 \cdot a_4 = 1 - \frac{1}{4} = \frac{3}{4};$$

$$a_7 = 1 - 0,5 \cdot a_6 = 1 - 0,5 \cdot \frac{5}{8} = 1 - \frac{5}{16} = \frac{11}{16}.$$

449.

$$1) 4; 4 \frac{1}{3}; 4 \frac{2}{3}; \dots a_1 = 4; d = a_2 - a_1 = \frac{1}{3};$$

$$a_4 = 4 + \frac{1}{3} \cdot 3 = 5; a_5 = 4 + \frac{1}{3} \cdot 4 = 5 \frac{1}{3};$$

$$2) 3 \frac{1}{2}; 3; 2 \frac{1}{2}; \dots$$

$$a_1 = 3 \frac{1}{2}; d = a_2 - a_1 = -\frac{1}{2};$$

$$a_4 = 3 \frac{1}{2} - \frac{1}{2} \cdot 3 = 2; a_5 = 2 - \frac{1}{2} = 1 \frac{1}{2};$$

$$3) 1; 1 + \sqrt{3}; 1 + 2\sqrt{3}; \dots$$

$$a_1 = 1; d = a_2 - a_1 = \sqrt{3};$$

$$a_4 = 1 + \sqrt{3} \cdot 3 = 1 + 3\sqrt{3}; a_5 = 1 + 3\sqrt{3} + \sqrt{3} = 1 + 4\sqrt{3};$$

$$4) \sqrt{2}; \sqrt{2} - 3; \sqrt{2} - 6; \dots$$

$$a_1 = \sqrt{2}; d = a_2 - a_1 = -3;$$

$$a_4 = \sqrt{2} - 3 \cdot 3 = \sqrt{2} - 9; a_5 = \sqrt{2} - 9 - 3 = \sqrt{2} - 12.$$

450.

$$\text{Найдем } a_{n+1} = -2(1 - (n+1)) = -2(-n) = 2n;$$

$$a_{n+1} - a_n = 2n - (-2(1-n)) = 2n + 2(1-n) = 2n + 2 - 2n = 2.$$

Т.к. $a_{n+1} - a_n$ не зависит от n , то a_n — арифметическая прогрессия.

451.

$$1) \underline{a_1 = 6; d = \frac{1}{2}}.$$

$$\text{Т.к. } a_5 = a_1 + 4d, \text{ то } a_5 = 6 + 4 \cdot \frac{1}{2} = 6 + 2 = 8;$$

$$2) \underline{a_1 = -3 \frac{1}{3}; d = -\frac{1}{3}}.$$

$$\text{Т.к. } a_7 = a_1 + 6d, \text{ то}$$

$$a_7 = -3 \frac{1}{3} + 6 \cdot \left(-\frac{1}{3}\right) = -3 \frac{1}{3} - 2 = -5 \frac{1}{3}.$$

452.

$$1) \underline{a_1 = -1; a_2 = 1};$$

$$d = a_2 - a_1 = 1 - (-1) = 2.$$

$$\text{Т.к. } S_{20} = \frac{2a_1 + 19d}{2} \cdot 20, \text{ то}$$

$$S_{20} = \frac{-2 + 38}{2} \cdot 20 = 360;$$

$$2) \underline{a_1 = 3; a_2 = -3};$$

$$d = a_2 - a_1 = -3 - 3 = -6.$$

$$\text{Т.к. } S_{20} = \frac{2a_1 + 19d}{2} \cdot 20, \text{ то}$$

$$S_{20} = \frac{6 - 114}{2} \cdot 20 = -1080.$$

453.

$$1) \underline{a_1 = -2; a_n = -60; n = 10}.$$

$$\text{Т.к. } S_{10} = \frac{a_1 + a_{10}}{2} \cdot 10, \text{ то}$$

$$S_{10} = (-2 - 60) \cdot 5 = -310;$$

$$2) \underline{a_1 = \frac{1}{2}; a_n = 25 \frac{1}{2}; n = 11}.$$

$$\text{Т.к. } S_{11} = \frac{a_1 + a_{11}}{2} \cdot 11, \text{ то}$$

$$S_{11} = \frac{\frac{1}{2} + 25 \frac{1}{2}}{2} \cdot 11 = 13 \cdot 11 = 143;$$

454.

$$a_1 = -38; d = 5; a_n = 12.$$

Т.к. $a_n = a_1 + (n-1)d$, то

$$12 = -38 + (n-1)5;$$

$$50 = (n-1) \cdot 5.$$

$$\text{Значит } n-1 = 10, n = 11;$$

$$S_{11} = \frac{-38+12}{2} \cdot 11 = -\frac{26}{2} \cdot 11 = -143.$$

Ответ: $S_{11} = -143$.

$$2) a_1 = -17; d = 3; a_n = 13.$$

Т.к. $a_n = a_1 + (n-1)d$, то

$$13 = -17 + (n-1) \cdot 3;$$

$$30 = (n-1) \cdot 3;$$

$$n-1 = 10;$$

$$n = 11 \quad S_{11} = \frac{-17+13}{2} \cdot 11 = -2 \cdot 11 = -22.$$

Ответ: $S_{11} = -22$.

455.

$$1) 3; 1; \frac{1}{3} \dots$$

$$q = b_2 : b_1 = \frac{1}{3}.$$

$$\text{Тогда } b_4 = 3 \cdot \left(\frac{1}{3}\right)^3 = \frac{1}{9}; b_5 = \frac{1}{9} \cdot \frac{1}{3} = \frac{1}{27};$$

$$2) \frac{1}{4}; \frac{1}{8}; \frac{1}{16} \dots$$

$$q = b_2 : b_1 = -\frac{1}{8} \cdot 4 = -\frac{1}{2}.$$

$$\text{Тогда } b_4 = \frac{1}{4} \cdot \left(-\frac{1}{2}\right)^3 = -\frac{1}{32}; b_5 = -\frac{1}{32} \cdot \left(-\frac{1}{2}\right) = \frac{1}{64};$$

$$3) 3; \sqrt{3}; 1 \dots$$

$$q = b_2 : b_1 = \sqrt{3} / 3 = \frac{1}{\sqrt{3}}.$$

$$\text{Тогда } b_4 = 3 \cdot \left(\frac{\sqrt{3}}{3}\right)^3 = 3 \cdot \frac{\sqrt{3}}{9} = \frac{\sqrt{3}}{3}; b_5 = \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{3} = \frac{1}{3};$$

4) если $5; -5\sqrt{2}; 10\dots$

$$q = b_2 : b_1 = -5\sqrt{2} : 5 = -\sqrt{2}.$$

$$\text{Тогда } b_4 = 5 \cdot (-\sqrt{2})^3 = -10\sqrt{2};$$

$$b_5 = -10\sqrt{2} \cdot (-\sqrt{2}) = 20.$$

456.

1) $-2; 4; -8;$

$$b_1 = -2; q = -2.$$

$$\text{Т.к. } b_n = b_1 \cdot q^{n-1},$$

$$b_n = -2 \cdot (-2)^{n-1} = (-2)^n;$$

2) $-\frac{1}{2}; 1; -2;$

$$b_1 = -\frac{1}{2}; q = -2.$$

$$\text{Т.к. } b_n = b_1 \cdot q^{n-1}, \text{ то}$$

$$b_n = -\frac{1}{2} \cdot (-2)^{n-1} = (-2)^{n-2}.$$

457.

1) $b_1 = 2; q = 2; n = 6.$

$$\text{Т.к. } b_6 = b_1 \cdot q^5, \text{ то}$$

$$b_6 = 2 \cdot 2^5 = 2 \cdot 32 = 64;$$

2) $b_1 = \frac{1}{8}; q = 5; n = 4.$

$$\text{Т.к. } b_4 = b_1 \cdot q^3, \text{ то}$$

$$b_4 = \frac{1}{8} \cdot 5^3 = \frac{125}{8}.$$

458.

1) $b_1 = \frac{1}{2}; q = -4; n = 5.$

$$\text{Т.к. } S_5 = \frac{b_1(1-q^5)}{1-q}, \text{ то}$$

$$S_5 = \frac{\frac{1}{2} \cdot (1 - (-4)^5)}{1 + 4} =$$

$$= \frac{1 + 1024}{2 \cdot 5} = 102,5$$

3) $b_1 = 10; q = 1; n = 6;$

$$S_6 = b_1 \cdot 6 = 10 \cdot 6 = 60;$$

2) $b_1 = 2; q = -\frac{1}{2}; n = 10;$

$$S_{10} = \frac{2 \cdot \left(1 - \left(-\frac{1}{2}\right)^5\right)}{1 + \frac{1}{2}} =$$

$$= \frac{4 \cdot \left(1 - \frac{1}{1024}\right)}{3} =$$

$$= \frac{4 \cdot 1023}{3 \cdot 1024} = \frac{341}{256} = 1 \frac{85}{256}$$

4) $b_1 = 5; q = -1; n = 9.$

$$\text{Т.к. } S_9 = \frac{b_1(1-q^9)}{1-q}, \text{ то}$$

$$S_9 = \frac{5 \cdot (1+1)}{1+1} = 5.$$

459.

1) 128; 64; 32; ... n = 5;

$$b_1 = 128; q = \frac{b_2}{b_1} = \frac{64}{128} = \frac{1}{2}.$$

$$\text{Тогда } S_5 = \frac{b_1(1-q^5)}{1-q} = \frac{128 \cdot \left(1 - \frac{1}{64}\right)}{1 - \frac{1}{2}} = \frac{2(128-2)}{1} = 2 \cdot 126 = 252;$$

2) 162; 54; 18; ... n = 5;

$$b_1 = 162; q = \frac{b_2}{b_1} = \frac{54}{162} = \frac{1}{3};$$

$$\begin{aligned} S_5 &= \frac{b_1(1-q^5)}{1-q} = \frac{162 \cdot \left(1 - \frac{1}{3^5}\right)}{1 - \frac{1}{3}} = -81 \cdot \left(1 - \frac{1}{243}\right) \cdot 3 = \\ &= \frac{-81 \cdot (-242) \cdot 3}{243} = 242 \end{aligned}$$

3) $\frac{2}{3}; \frac{1}{2}; \frac{3}{8}; \dots n = 5;$

$$b_1 = \frac{2}{3}; q = \frac{b_2}{b_1} = \frac{1 \cdot 3}{2 \cdot 2} = \frac{3}{4};$$

$$\begin{aligned} S_5 &= \frac{b_1(1-q^5)}{1-q} = \frac{\frac{2}{3} \cdot \left(1 - \left(\frac{3}{4}\right)^5\right)}{1 - \frac{3}{4}} = \frac{2 \cdot 4 \cdot \left(1 - \frac{243}{1024}\right)}{3} = \frac{8 \cdot 781}{3 \cdot 1024} = \\ &= \frac{781}{384} = 2 \frac{13}{384}; \end{aligned}$$

4) $\frac{3}{4}; \frac{1}{2}; \frac{1}{3}; \dots n = 4;$ $b_1 = \frac{3}{4}; q = \frac{b_2}{b_1} = \frac{1 \cdot 4}{2 \cdot 3} = \frac{2}{3};$

$$S_4 = \frac{b_1(1-q^4)}{1-q} = \frac{\frac{3}{4} \cdot \left(1 - \left(\frac{2}{3}\right)^4\right)}{1 - \frac{2}{3}} = \frac{3 \cdot 3 \cdot \left(1 - \frac{16}{81}\right)}{4} = \frac{9 \cdot 65}{81 \cdot 4} = \frac{65}{36} = 1 \frac{29}{36}.$$

460.

1) $\frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \dots$

$$q = b_2 : b_1 = -\frac{1}{4} : \frac{1}{2} = -\frac{1}{2}.$$

Т.к. $\left| -\frac{1}{2} \right| < 1$, то b_n — бесконечно убывает

$$\text{и } S = \frac{\frac{1/2}{1 + 1/2}}{\frac{2}{2 \cdot 3} = \frac{1}{3}};$$

2) $-1; \frac{1}{4}; \frac{1}{16}; \dots$

$$q = b_2 : b_1 = \frac{1}{4} : -1 = -\frac{1}{4}.$$

Т.к. $\left| -\frac{1}{4} \right| < 1$, то b_n — бесконечно убывает

$$\text{и } S = \frac{-1}{1 + 1/4} = \frac{-1}{5/4} = \frac{-4}{5}.$$

461.

$$n = 1, a_3 = \frac{a_1 + a_2}{2} = \frac{-1 + 3}{2} = 1;$$

$$n = 2, a_4 = \frac{a_2 + a_3}{2} = \frac{3 + 1}{2} = 2;$$

$$n = 3, a_5 = \frac{a_3 + a_4}{2} = \frac{1 + 2}{2} = \frac{3}{2}.$$

462.

Т.к. $a_8 = a_1 + 7d$, то

$$23 \frac{1}{2} = 2 \frac{1}{2} + 7d \text{ и } d = 3.$$

463.

1) $\underline{a_1 = 5; a_3 = 15.}$

Т.к. $a_3 = a_1 + 2d$, то

$$15 = 5 + 2d;$$

$$d = 5;$$

$$a_2 = 10;$$

$$a_3 = 15; a_4 = 20; a_5 = 25;$$

Ответ: 5; 10; 15; 20; 25.

2) $\underline{a_3 = 8; a_5 = 2.}$

Т.к. $a_5 = a_3 + 2d$, то

$$2 = 8 + 2d;$$

$$d = -3;$$

$$a_4 = 5; a_2 = 11; a_1 = 14.$$

Ответ: 14; 11; 8; 5; 2.

464.

Чтобы a_1, a_2, a_3 были членами арифметической прогрессии,

$$\text{надо, чтобы } a_2 = \frac{a_1 + a_3}{2},$$

$$\text{тогда } a_2 = \frac{-10 + 5}{2} = -\frac{5}{2} = -2,5.$$

465.

1) $a_{13} = 28; a_{20} = 38.$

Т.к. $a_{20} = a_{13} + 7d$, то

$$38 = 28 + 7d.$$

Значит $10 = 7d$

$$\text{и } d = 1 \frac{3}{7};$$

$$a_{19} = a_{20} - d;$$

$$a_{19} = 38 - 1 \frac{3}{7} = 36 \frac{4}{7}.$$

Т.к. $a_{13} = a_1 + 12d$, то

$$a_1 = 28 - 12 \cdot 1 \frac{3}{7} =$$

$$= 28 - 12 - 5 \frac{1}{7} = 10 \frac{6}{7}.$$

Ответ: $a_1 = 10 \frac{6}{7}; a_{19} = 36 \frac{4}{7}.$

2) $a_{18} = -6; a_{20} = 6.$

Т.к. $a_{19} = \frac{a_{18} + a_{20}}{2}$, то

$$a_{19} = \frac{-6 + 6}{2} = 0.$$

Отсюда $d = a_{20} - a_{19} = 6.$

Т.к. $a_{20} = a_1 + 19d$, то

$$6 = a_1 + 19 \cdot 6;$$

$$a_1 = 6 - 19 \cdot 6 = -108.$$

Ответ: $a_1 = -108; a_{19} = 0.$

466.

1) Для того, чтобы это была арифметическая прогрессия надо, чтобы

$$\frac{x+2}{2} = \frac{(3x+2x-1)}{2} = \frac{5x-1}{2};$$

$$x+2 = 5x-1;$$

$$4x = 3;$$

$$x = \frac{3}{4};$$

2) Для того, чтобы это была арифметическая прогрессия надо, чтобы

$$2 = \frac{3x^2 + 11x}{2};$$

$$3x^2 + 11x - 4 = 0.$$

Решим:

$$x_1 = \frac{2}{6} = \frac{1}{3}; x_2 = \frac{-24}{6} = -4.$$

Ответ: $\frac{1}{3}; -4.$

467.

1) $\underline{b_1 = \sin(\alpha + \beta); b_2 = \sin\alpha \cdot \cos\beta; b_3 = \sin(\alpha - \beta)}$.

Если $b_2 = \frac{b_1 + b_3}{2}$, то b_1, b_2, b_3 , — арифметическая прогрессия;

$$\sin\alpha \cdot \cos\beta = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2} = \frac{2\sin\alpha \cdot \cos\beta}{2} = \sin\alpha \cdot \cos\beta.$$

Верно.

2) $\underline{b_1 = \cos(\alpha + \beta); b_2 = \cos\alpha \cdot \cos\beta; b_3 = \cos(\alpha - \beta)}$.

Если $b_2 = \frac{b_1 + b_3}{2}$, то b_1, b_2, b_3 — арифметическая прогрессия

$$\cos\alpha \cdot \cos\beta = \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2} = \frac{2\cos\alpha \cdot \cos\beta}{2} = \cos\alpha \cdot \cos\beta.$$

Верно.

3) $\underline{b_1 = \cos 2\alpha; b_2 = \cos^2\alpha; b_3 = 1}$.

Если $b_2 = \frac{b_1 + b_3}{2}$, то b_1, b_2, b_3 — арифметическая прогрессия

$$\begin{aligned} \cos^2\alpha &= \frac{\cos 2\alpha + 1}{2} = \frac{\cos^2\alpha - \sin^2\alpha + \cos^2\alpha + \sin^2\alpha}{2} = \\ &= \frac{2\cos^2\alpha}{2} = \cos^2\alpha. \end{aligned}$$

Верно.

4) $\underline{b_1 = \sin 5\alpha; b_2 = \sin 3\alpha \cos 2\alpha; b_3 = \sin\alpha}$.

Нужно $b_2 = \frac{b_1 + b_3}{2}$, чтобы b_1, b_2, b_3 были арифметической прогрессией

$$\sin 3\alpha \cdot \cos 2\alpha = \frac{\sin 5\alpha + \sin\alpha}{2}; \quad \sin 3\alpha \cdot \cos 2\alpha = \frac{2\sin 3\alpha \cdot \cos 2\alpha}{2};$$

$$\sin 3\alpha \cdot \cos 2\alpha = \sin 3\alpha \cdot \cos 2\alpha. \text{ Верно.}$$

468.

$$d = a_2 - a_1 = 7 - 5 = 2.$$

$$\text{Тогда } S_n = \frac{2a_1 + (n-1)d}{2} \cdot n;$$

$$252 = \frac{10 + (n-1) \cdot 2}{2} \cdot n;$$

$$504 = (8 + 2n) \cdot n; \quad 252 = (4 + n) \cdot n; \quad n^2 + 4n - 252 = 0. \text{ Решим:}$$

$$n_1 = 14; \quad n_2 = -32 - \text{не натуральное число.}$$

Ответ: 14.

469.

1) $\underline{a_1 = 40, n = 20, S_{20} = -40.}$

Т.к. $S_{20} = \frac{a_1 + a_{20}}{2} \cdot 20$, то

$$-40 = (a_1 + a_{20}) \cdot 10.$$

Значит $-4 = (40 + a_{20})$;

$$a_{20} = -44.$$

Т.к. $d = \frac{a_{20} - a_1}{19}$, то

$$d = \frac{-44 - 40}{19} = \frac{-84}{19} = -4 \frac{8}{19}.$$

Ответ: $a_{20} = -44, d = -4 \frac{8}{19}.$

2) $\underline{a_1 = \frac{1}{3}, n = 16, S_{16} = -10 \frac{2}{3}.}$

Т.к. $S_{16} = \frac{a_1 + a_{16}}{2} \cdot 16$, то

$$S_{16} = \frac{2a_1 + 15d}{2} \cdot 16.$$

Значит $-10 \frac{2}{3} = \left(\frac{2/3 + 15d}{2} \right) \cdot 16$;

$$-10 \frac{2}{3} = 2/3 + 15d \cdot 8$$

$$-16 = 120d$$

$$d = -\frac{2}{15}.$$

Т.к. $a_{16} = a_1 + 15d$, то

$$a_{16} = \frac{1}{3} + 15 \cdot \left(-\frac{2}{15} \right) = \frac{1}{3} - 2 = -1 \frac{2}{3}.$$

Ответ: $a_{16} = -1 \frac{2}{3}, d = -\frac{2}{15}.$

470.

1) Т.к. $b_9 = b_1 \cdot q^8$,
то $b_9 = 4 \cdot (-1)^8 = 4.$

2) Т.к. $b_7 = b_1 \cdot q^6$,
то $b_7 = 1 \cdot (\sqrt{3})^6 = 27.$

471.

1) $b_2 = \frac{1}{2}, b_7 = 16$;

$$b_7 = b_2 \cdot q^5,$$

тогда

$$16 = \frac{1}{2} \cdot q^5, q^5 = 32, q = 2.$$

Т.к. $b_5 = b_2 \cdot q^3$, то

$$b_5 = \frac{1}{2} \cdot 8 = 4;$$

2) $b_3 = -3, b_6 = -81$;

$$b_6 = b_3 \cdot q^3,$$

тогда

$$-81 = -3 \cdot q^3;$$

$$q^3 = 27, \text{ значит } q = 3.$$

Т.к. $b_5 = b_3 \cdot q^2$, то

$$b_5 = -3 \cdot 9 = -27;$$

$$3) \underline{b_2 = 4, b_4 = 1.}$$

Г.к. $b_4 = b_2 \cdot q^2$, то

$$1 = 4 \cdot q^2;$$

$$q_{1,2} = \pm \frac{1}{2}.$$

Если $q = \frac{1}{2}$, то

$$b_5 = b_4 \cdot q,$$

$$\text{имеем: } b_5 = 1 \cdot \frac{1}{2} = \frac{1}{2}.$$

Если $q = -\frac{1}{2}$,

то $b_5 = b_4 \cdot q$, имеем: $b_5 = -\frac{1}{2}$.

Ответ: $b_5 = \frac{1}{2}$ или $b_5 = -\frac{1}{2}$.

$$4) \underline{b_2 = -\frac{1}{5}, b_6 = -\frac{1}{125}.}$$

Г.к. $b_6 = b_2 \cdot q^4$, то

$$-\frac{1}{125} = -\frac{1}{5} \cdot q^4$$

$$q^4 = \frac{1}{25}, q_{1,2} = \pm \frac{1}{5}$$

Если $q = \frac{1}{5}$,

$$\text{то } b_5 = -\frac{1}{5} \cdot \frac{1}{5} = -\frac{1}{25}.$$

Если $q = -\frac{1}{5}$,

$$\text{то } b_5 = \left(-\frac{1}{5}\right) \cdot \left(-\frac{1}{5}\right) = \frac{1}{25}.$$

Ответ: $b_5 = -\frac{1}{25}$ или $b_5 = \frac{1}{25}$.

472.

Чтобы $b_1; b_2; b_3$ – были членами геометрической прогрессии, необходимо, чтобы

$$b_2^2 = b_1 \cdot b_3, \text{ значит,}$$

$$b_2^2 = 36, b_2 = 6 \text{ или } b_2 = -6.$$

473.

1) $\underline{b_n = 5^{n+1}}$ – последовательность;

$b_1 = 25; b_2 = 125, q = \frac{b_2}{b_1} = \frac{125}{25} = 5 > 1$, не является бесконечно

убывающей;

2) $\underline{b_n = (-4)^{n+2}}$ – последовательность;

$b_1 = -64; b_2 = 256, q = \frac{256}{-64} = -4, |q| > 1$, не является бесконечно

убывающей;

3) $\underline{b_n = \frac{10}{7^n}}$ – последовательность;

$b_1 = \frac{10}{7}; b_2 = \frac{b_2}{b_1} = \frac{10}{49} \cdot \frac{7}{10} = \frac{1}{7} < 1$, b_n – бесконечно убывает;

$$4) \underline{b_n = \frac{50}{3^{n+3}}} - \text{последовательность};$$

$$b_1 = -\frac{50}{81}; b_2 = \frac{b_2}{b_1} = \frac{50}{243} \cdot \frac{81}{50} = \frac{1}{3} < 1,$$

b_n – бесконечно убывает.

474.

$$1) b_2 = -81, S_2 = 162.$$

Т.к. $S_2 = b_1 + b_2$, то

$$162 = b_1 - 81, \text{отсюда}$$

$$b_1 = 243;$$

$$q = \frac{b_2}{b_1} = \frac{-81}{243} = -\frac{1}{3};$$

$$|q| = \left| -\frac{1}{3} \right| < 1, \text{значит, } b_n \text{ бесконечно убывает};$$

$$2) b_2 = 33, S_2 = 67.$$

Т.к. $S_2 = b_1 + b_2$, то

$$67 = b_1 + 33, b_1 = 34, q = \frac{b_2}{b_1} = \frac{33}{34} < 1,$$

значит, b_n бесконечно убывает.

3) Пусть $b_1 + b_3 = 130$; $b_1 - b_3 = 120$, запишем систему

$$\begin{cases} b_1 + b_3 = 130 \\ b_1 - b_3 = 120 \end{cases} \begin{cases} 2b_1 = 250 \\ 2b_3 = 10 \end{cases} \begin{cases} b_1 = 125 \\ b_3 = 5 \end{cases};$$

Т.к. $b_3 = b_1 \cdot q^2$, то

$$5 = 125 \cdot q^2, q^2 = \frac{1}{25}, q = \pm \frac{1}{5},$$

$$\left| \pm \frac{1}{5} \right| < 1, \text{значит, } b_n \text{ бесконечно убывает};$$

4) Пусть $b_2 + b_4 = 68$; $b_2 - b_4 = 60$, решим систему

$$\begin{cases} b_2 + b_4 = 68 \\ b_2 - b_4 = 60 \end{cases} \begin{cases} 2b_2 = 128 \\ 2b_4 = 8 \end{cases} \begin{cases} b_2 = 64 \\ b_4 = 4 \end{cases};$$

Т.к. $b_4 = b_2 \cdot q^2$, то

$$4 = 64 \cdot q^2;$$

$$q^2 = \frac{1}{16} \quad q = \pm \frac{1}{4};$$

$$\left| \pm \frac{1}{4} \right| < 1 \text{ значит, } b_n \text{ бесконечно убывает.}$$

475.

Пусть n – номер дня, a_n – количество минут в n день.

Т.к. $a_n = a_1 + (n - 1)d$, то

$$40 = 5 + (n - 1) \cdot 5;$$

$$35 = (n - 1) \cdot 5;$$

$$n - 1 = 7 \text{ значит } n = 8.$$

Ответ: Восьмой день от среды – среда.

476.

Решим систему относительно a_1 и d :

$$\begin{cases} a_1 + a_2 + a_3 = 15 \\ a_1 a_2 a_3 = 80 \end{cases}, \quad \begin{cases} a_1 + a_1 + d + a_1 + 2d = 15 \\ a_1(a_1 + d)(a_1 + 2d) = 80 \end{cases}$$

$$3a_1 + 3d = 15;$$

$$a_1 + d = 5;$$

$$a_1 = 5 - d.$$

Подставим во второе уравнение системы:

$$(5 - d)(5 - d + d)(5 - d + 2d) = 80;$$

$$5(5 - d)(5 + d) = 80;$$

$$25 - d^2 = 16;$$

$$d^2 = 9. \text{ Значит,}$$

$$d = 3 \text{ или } d = -3.$$

$$\text{Тогда } a_1 = 5 - 3 = 2 \text{ или } a_1 = 5 + 3 = 8.$$

Ответ: $d = 3$, $a_1 = 2$; $d = -3$, $a_1 = 8$.